Homework 9: Control

EECS/BioE/MechE C106A/206A
Introduction to Robotics

Due: November 13, 2023

Problem 1: Cat and mouse

Kaylene's cat Zena loves chasing wand toys, but when her roommates are out working on robots she doesn't have anyone to play with her :(Kaylene wants to build a wand toy that can be controlled autonomously when she isn't home. In this problem, we'll derive the Lagrangian of the toy in terms of the angles θ and ϕ so that we can calculate the torques that should be applied to make the toy follow a path that's fun for Zena to chase.

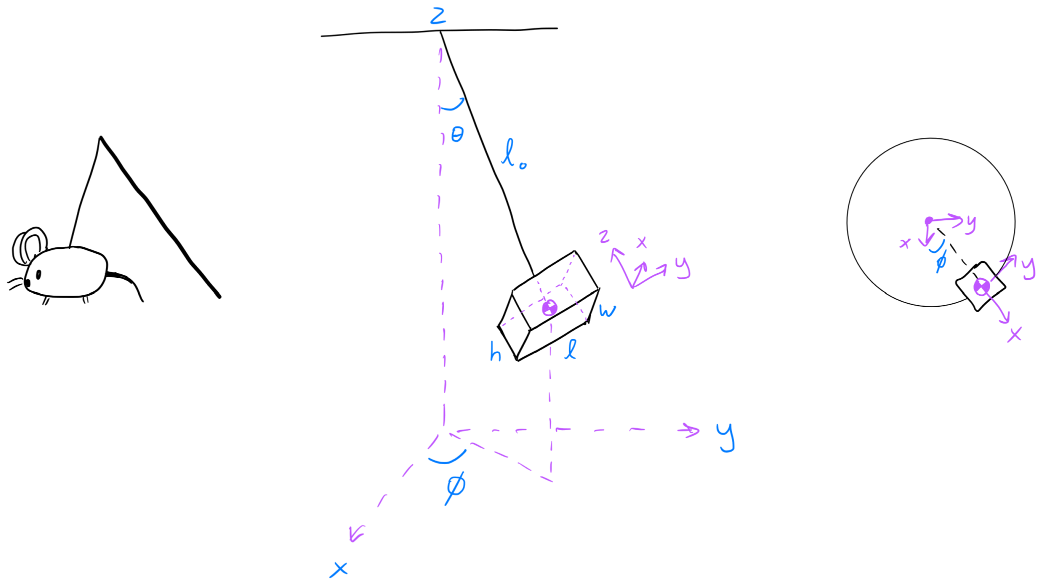

Figure 1: Left: The wand toy with a stuffed mouse. Middle: Model of the toy. Right: top-view diagram of the toy model.

(a) Find the Lagrangian of the toy if we model the dangling mouse as a point mass of mass m. Use θ and ϕ for the generalized coordinates, and let the height of the top of the string be 0 for the potential energy.

(b) After implementing a controller based on the Lagrangian for part (a), we find that it doesn't work as well as we hoped. To obtain a more accurate model, we decide to model the mouse as a homogeneous box instead of a point mass. What is the Lagrangian for this system? Note that the projection of the string that anchors the box and the x-axis of the box onto the xy plane are aligned (i.e. the x-axis of the body frame always points away from the anchor point - see the top-view diagram).

Problem 2: OSIRIS-RExploring

On 24 September, the OSIRIS-REx satellite safely returned samples from the asteroid Bennu to Earth! Before collecting its samples, it spent 2 years orbiting Bennu and mapping its surface. If we model the satellite and asteroid as point masses, we can write the normalized equations of motion of the satellite as

$$\ddot{r} = r\dot{\theta}^2 - \frac{k}{r^2} + u_1 \tag{1}$$

$$\ddot{\theta} = -2\frac{\dot{\theta}}{r}\dot{r} + \frac{1}{r}u_2 \tag{2}$$

where r is the distance from the satellite to the asteroid, θ is the satellite's angular position, k is a normalization constant, and u_1 and u_2 are control inputs from the satellite's thrusters.

(a) Without any thruster input ($u_1 = u_2 = 0$), the satellite achieves a circular orbit with $r(t) = p$ and $\theta(t) = \omega t$. Linearize the system about this orbit, noting that for this orbit we can replace k with $p^3\omega^2$ (from the first equation).

(b) Characterize the stability of the linearized system when the thrusters are not firing. What does this mean for the satellite's orbit? *Hint: you may use the eigenvals() function in Sympy.*

(c) Is the linearized system completely controllable?

Problem 3: SpaceMax

Max, who is also passionate about rocketeering, has decided to start his own, rival company, SpaceMax. He designs the rocket depicted in Figure 2.

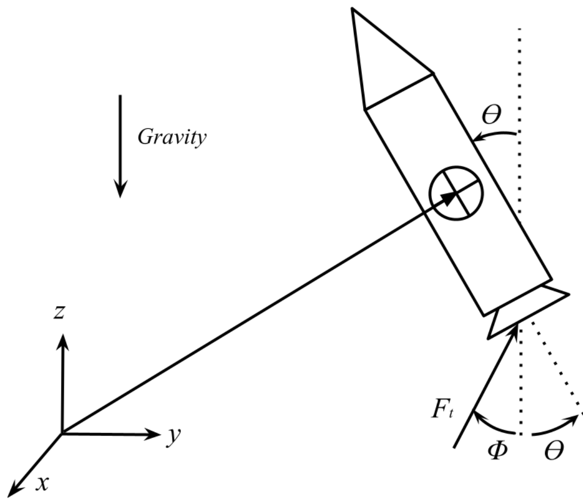

Figure 2: Gimbaled Rocket

The rocket is powered by a gimbaled engine, which rotates by an angle Φ with respect to spatial z axis to control the angle at which the thrust force, F_t, is applied. The rocket has constant mass m and inertia I about the center of mass. Note that since the rocket is planar, $x(t) = 0$ for all t.

(a) Write down a vector of generalized coordinates for the system. (*Hint: this should be a vector of length 3.*)

(b) Find the Lagrangian $L = T - V$ of the system.

(c) Find the rocket's equations of motion in terms of a vector of generalized forces Υ.

(d) Calculate Υ_y, the component of the external force in the y-direction. Assume that the thrust force F_t is the only external force applied to the system.

(e) Calculate Υ_z, the component of the external force in the z-direction. Assume that the thrust force F_t is the only external force applied to the system.

(f) Max wants to keep his rocket stable about $z = z_d$. This suggests trying to drive the error $e = z_d - z$ to 0. Prove that for any system with $e \in \mathbb{R}^n$, if e evolves according to the following ODE,

$$0 = K_p e + K_d \dot{e} + c\ddot{e} \tag{3}$$

where $c \in \mathbb{R}$ is a constant, we can choose $K_p, K_d \in \mathbb{R}^{n\times n}$ such that $\lim_{t\to\infty} e(t) = 0$. *Hint: Choose convenient K_p and K_d matrices and examine an arbitrary row of the ODE.*

(g) Assuming a fixed gimbal angle Φ, find an expression for the thrust force F_t that gives error dynamics of:

$$0 = K_p e + K_d \dot{e} + \ddot{e} \tag{4}$$

Where $e = z_d - z$. You may leave your answer in terms of gains K_p, K_d, and constant gimbal angle Φ. You may assume that the thrust vector has a positive z component. Using the result of part (f), we can conclude that the rocket will stably hover at z_d!