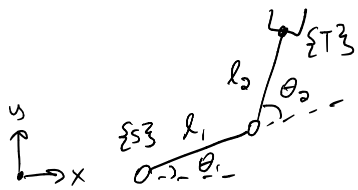
Discussion 3: Forward kinematics



Goal: find the map $g_{ST}(\theta_1, \theta_2)$

Try 1: work directly

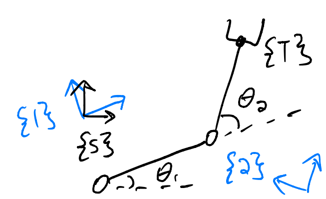
rotation: $R_{ST} = \begin{bmatrix} \cos(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) & 0 \\ \sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) & 0 \\ 0 & 0 & 1 \end{bmatrix}$

translation: $p = \begin{bmatrix} l_1 \cos\theta_1 + l_2 \cos(\theta_1+\theta_2) \\ l_1 \sin\theta_1 + l_2 \sin(\theta_1+\theta_2) \\ 0 \end{bmatrix}$

$$g_{ST}(\theta_1, \theta_2) = \begin{bmatrix} R_{ST} & p \\ 0 & 1 \end{bmatrix}$$

Try 2: composition of homogeneous transforms

$T_{S1} = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$



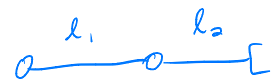
$T_{12} = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & l_1 \\ \sin\theta_2 & \cos\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$T_{1T} = \begin{bmatrix} 1 & 0 & 0 & l_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$$g_{ST}(\theta_1, \theta_2) = T_{S1} T_{12} T_{2T}$$

Try 3: "product of exponentials" (PoE)

let $\theta_1 = \theta_2 = 0$



$$g_{ST}(0) = \begin{bmatrix} 1 & 0 & 0 & \ell_1 + \ell_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now, fix θ_1 but allow θ_2 to move.

$$g_{ST}(\theta_1) = e^{\hat{\xi}_2 \theta_2} g_{ST}(0)$$

Allowing both to move:

$$g_{ST}(\theta_1, \theta_2) = e^{\hat{\xi}_1 \theta_1} e^{\hat{\xi}_2 \theta_2} g_{ST}(0)$$

$$\xi_1 : \omega_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad v_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\xi_2 : \omega_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad v_2 = -\omega \times q_2, \quad \text{let } q_2 = \begin{bmatrix} \ell_1 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow v_2 = -(1\hat{k}) \times \ell_1 \hat{i} = -\ell_1 \hat{j} = \begin{bmatrix} 0 \\ -\ell_1 \\ 0 \end{bmatrix}$$