LOST 928

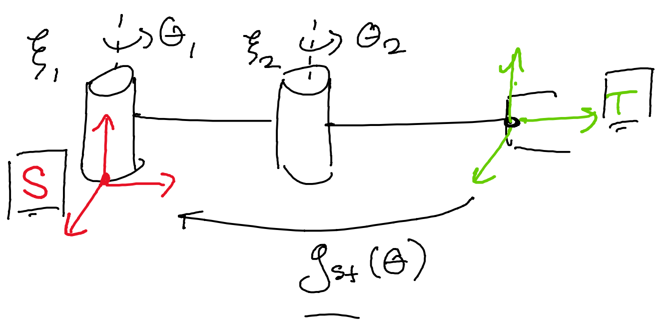
Wednesday, September 28, 2022 5:09 PM

LOST SECTION 9/28

Agenda:

0. Manipulator Workspace ? (or save extra time for MT)
1. PK Subproblems
2. Inverse Kinematics

PK SUBPROBLEMS + IK:



FORWARD KIN:

$$g_{st}(\theta) = e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} g_{st}(0)$$

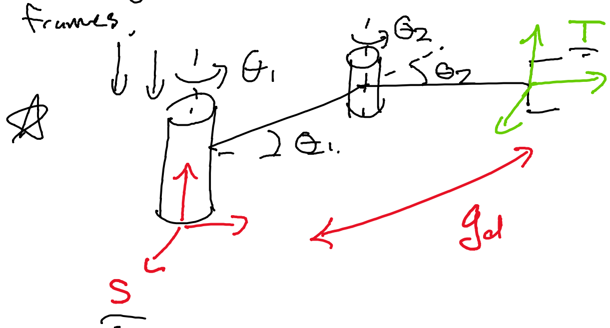
$\theta = \begin{bmatrix}\theta_1 \\ \theta_2\end{bmatrix}$

$e^{\hat{\xi}_1\theta_1}$: rotate 1st joint

$e^{\hat{\xi}_2\theta_2}$: rotate 2nd joint

$g_{st}(0)$: ↳ Convert btwn. T & S in ref. config

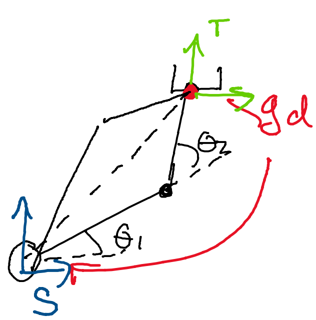
- Do this in REVERSE!
- let g_d be the transf. we WANT between our S + T frames.



$$\boxed{e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} g_{st}(0) = g_d}$$

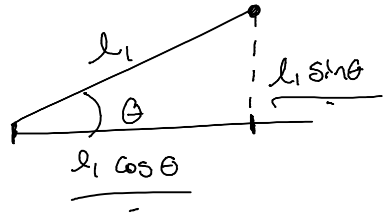
"INVERSE KIN."

⇒ Solve this eqn. for θ_1 & θ_2.



Problems:

- Soln. for θ_1 + θ_2 not unique!
  ⇒ MANY diff. solns!
- VERY nonlinear, HARD TO SOLVE!



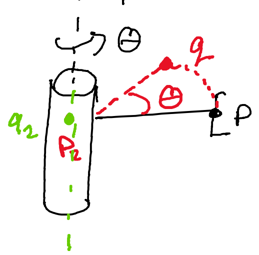
- Split up OVERALL problem into a set of simpler ones that we KNOW how to solve!

- Break down LARGE robots into simpler ones!
- "PADEN-KAHAN SUBPROBLEMS": "PK"
- Prismatic / Screw: find ACTUAL formula!
- For REVOLUTE: leave in terms of PK solns.

- 2 prismatic joints $\|$ $\Rightarrow$ ∞ number of solns.

SUBPROBLEM 1: (PK1)

- simplest possible robot:



Ignore $\theta \pm 2\pi \pm 4\pi \ldots$

$\Rightarrow$ Can find a twist, ξ, for this simple revolute joint!

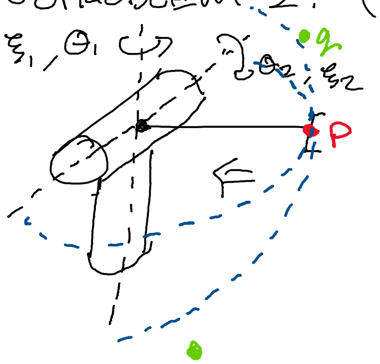
$$\boxed{e^{\hat{\xi}\theta} p = q} \quad \Big\} \text{ PK1}$$

transf. of rotating a single revolute joint!

$\Rightarrow$ we KNOW how to solve this for θ.

☆ - MAX. of 1 soln! ☆

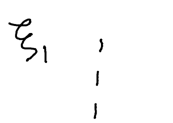
SUBPROBLEM 2: (PK2)



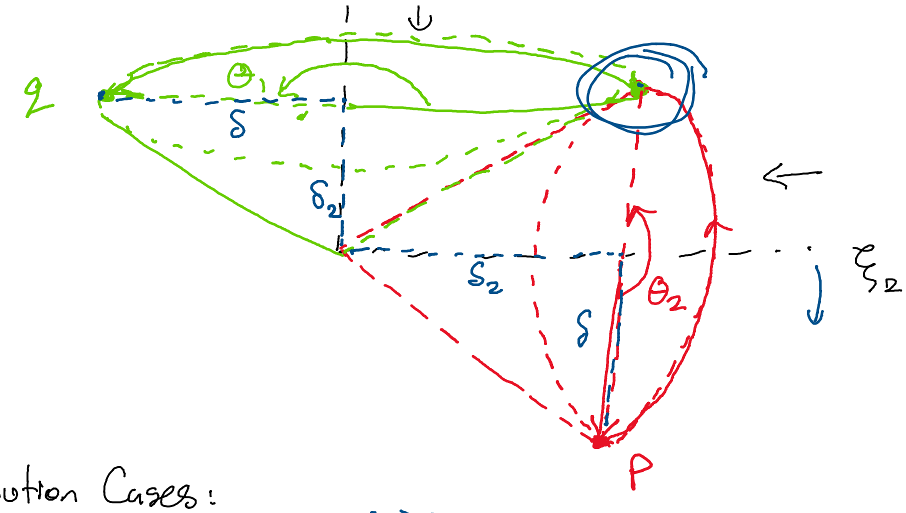
$\Rightarrow$ Two revolute joints whose AXES intersect!!

$$\boxed{e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} p = q} \quad \text{"PK2"}$$

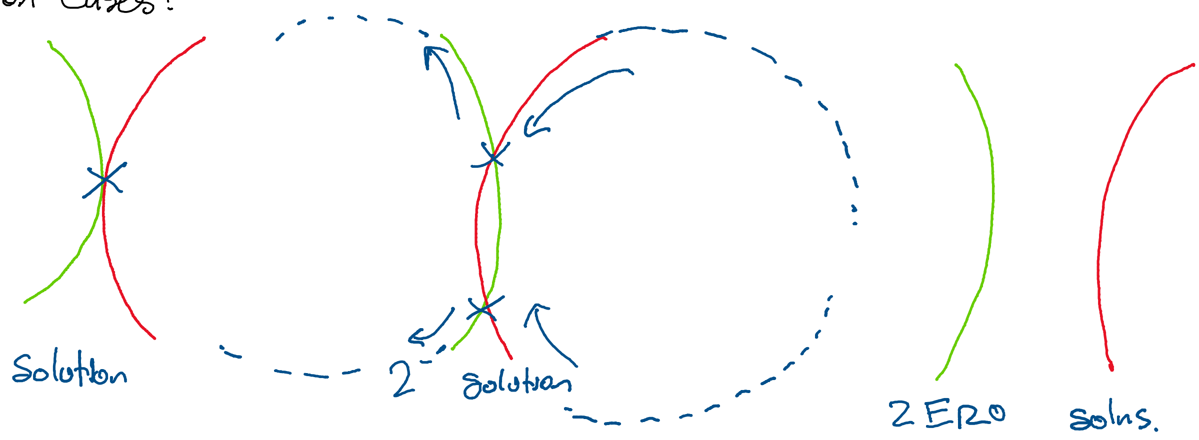
$\Rightarrow$ PK2 solves this prob. for θ_1 AND θ_2.

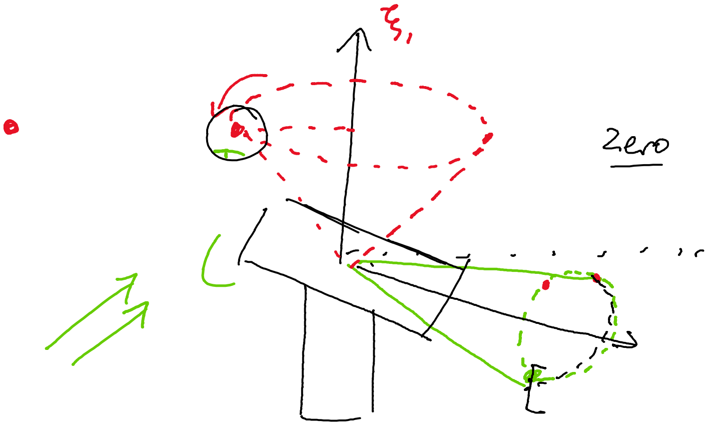


$$e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} p = q$$

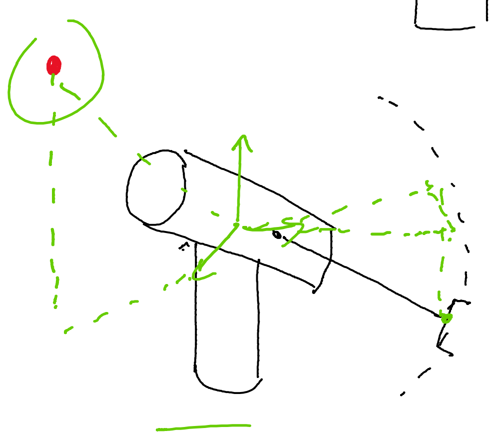


Solution Cases:

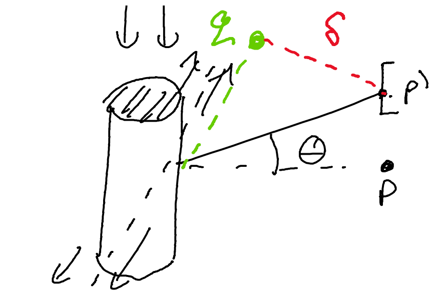




zero solns.



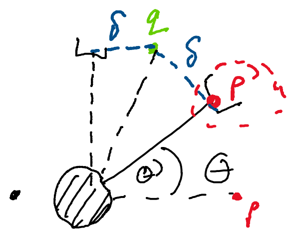
SUBPROBLEM 3: (PK3) ✫ NOT FOR PRISMATIC/ SCREW JOINTS ✫



$$\boxed{\| e^{\hat{\xi}\theta} \underline{p} - q \| = \delta} \quad PK3$$

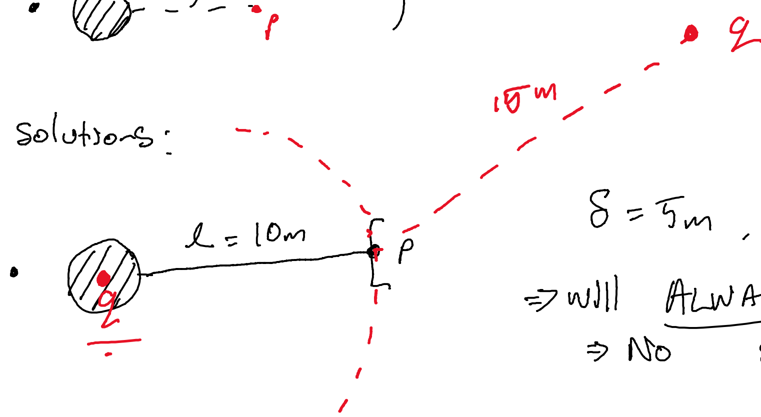
⇒ How do we get θ s.t. p' is distance δ away from q?

- How many solns??



Symmetry gives a <u>MAX</u> of 2 solutions!!

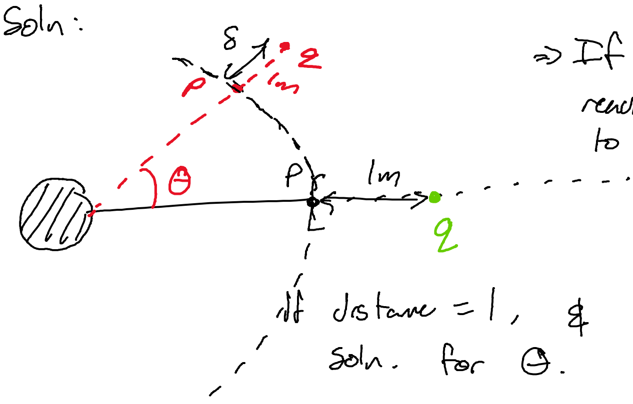
<u>NO</u> solutions:



$\delta = 5m$, $\delta = 10 \Rightarrow \infty$ solns.

⇒ will <u>ALWAYS</u> be 10m away!
⇒ No solution!!

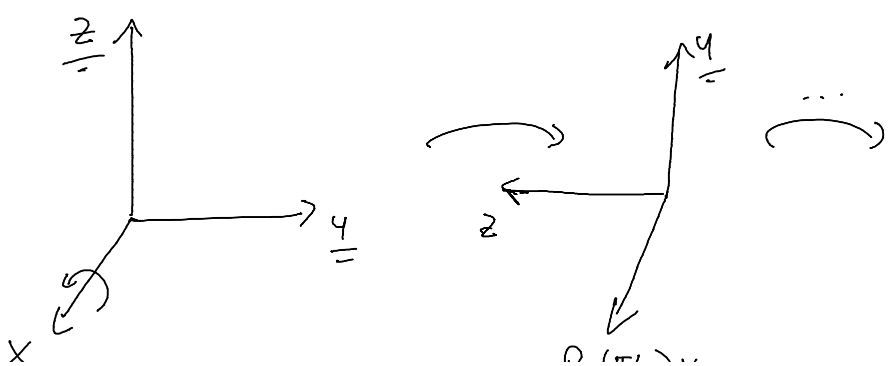
<u>ONE</u> Soln:



⇒ If q is <u>BEYOND</u> maximum reach & δ is appr. chosen to be the <u>distance</u> to the circle, we have 1 soln.

If distance = 1, & $\delta = 1$, we have a <u>single</u> soln. for θ.

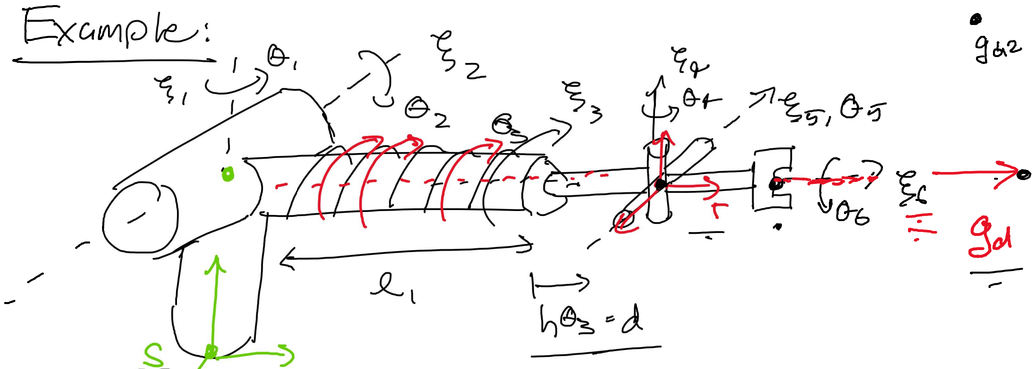
$R_z(\pi)\, \underline{R_x(\pi/2)}$



$R_x(\pi/2)X$

⇒ When we apply $R_x(\pi/2)$ to something in our initial frame, it ROTATES the vecs. by $\pi/2$ in that frame.

Example:



Joint 3 is a SCREW joint with pitch (h)!

Solution: Idea: treat screw joint like it's prismatic!

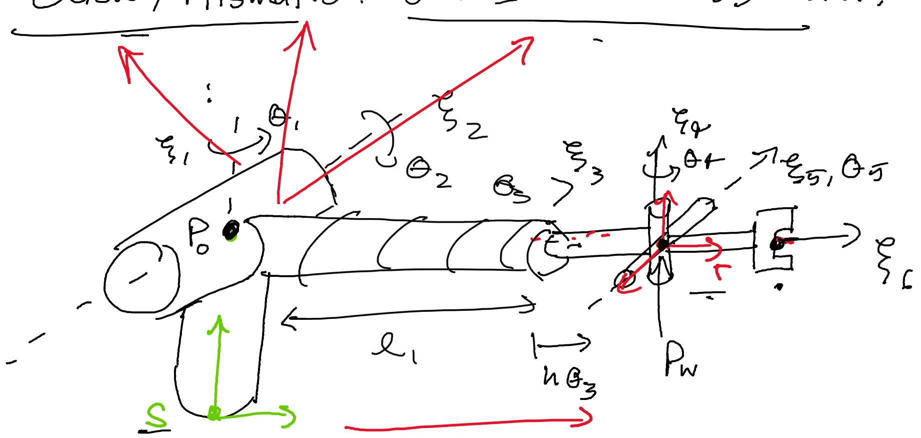
Step 1: P.O.E: $e^{\hat\xi_1\theta_1} e^{\hat\xi_2\theta_2} e^{\hat\xi_3\theta_3} e^{\hat\xi_4\theta_4} e^{\hat\xi_5\theta_5} e^{\hat\xi_6\theta_6} g_{st}(0) = g$

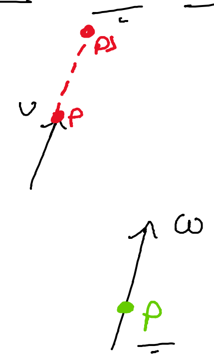
take inverse!

$$e^{\hat\xi_1\theta_1} e^{\hat\xi_2\theta_2} e^{\hat\xi_3\theta_3} e^{\hat\xi_4\theta_4} e^{\hat\xi_5\theta_5} e^{\hat\xi_6\theta_6} = g_d\, g_{st}^{-1}(0) = g$$

↑ know this matrix!

Screw/Prismatic: SOLVE FOR THESE FIRST!





⇒ If we rotate a point ALREADY on the axis, nothing happens!!

$$e^{\hat\xi_1\theta_1} e^{\hat\xi_2\theta_2} e^{\hat\xi_3\theta_3} e^{\hat\xi_4\theta_4} e^{\hat\xi_5\theta_5} e^{\hat\xi_6\theta_6} p_w = g\, p_w$$

Can't yet invert, don't know what they are!!

$$e^{\hat\xi_1\theta_1} e^{\hat\xi_2\theta_2} e^{\hat\xi_3\theta_3} p_w = g\, p_w$$

"g_d"

"Subtract a point, take the norm"

$e^{\hat\xi_1\theta_1} e^{\hat\xi_2\theta_2} e^{\hat\xi_3\theta_3}$

$$\underline{e\ \ e}\ \ e\ \ p_w - \underline{p_o} = \underline{g\,p_w} - p_o$$

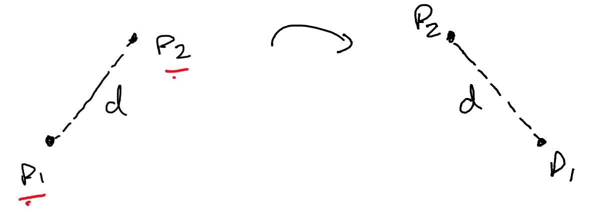
$$\underline{e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2}} e^{\hat{\xi}_3\theta_3} p_w - \underline{e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2}}\, \underline{p_o} = \underline{g\,p_w - p_o}$$

diff. btwn pts

$$\left\| \underline{e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2}} \left(e^{\hat{\xi}_3\theta_3} p_w - p_o \right) \right\| = \| g\,p_w - p_o \|$$

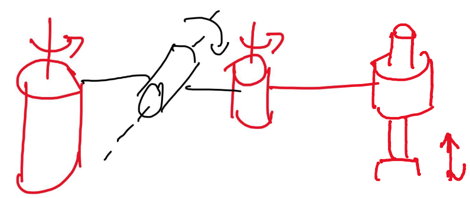
diff. btwn. two points!

"Rigid motions preserve distance between points"

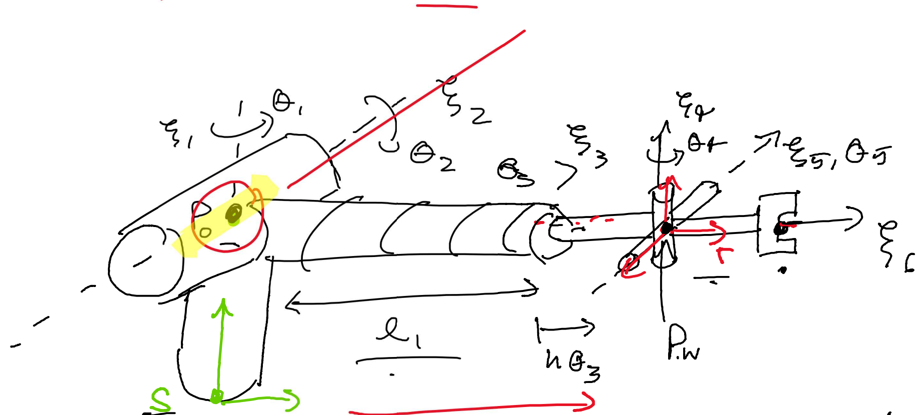


$$\underline{\| e^{\hat{\xi}_3\theta_3} p_w - p_o \|} = \underline{\| g\,p_w - p_o \|}$$

} B/C $e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2}$ is a rigid, length. pres. transf.



This is NOT SP3!!

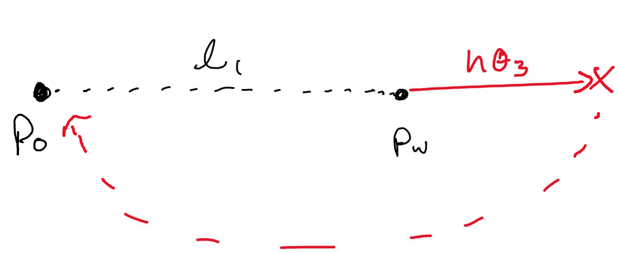


we know this!

$$\| \underbrace{e^{\hat{\xi}_3\theta_3} p_w} - p_o \| = \| g\,p_w - p_o \|$$

Wrist pos. after moving along ξ_3 by $h\theta_3$

p_o = point on ξ_1 & ξ_2.



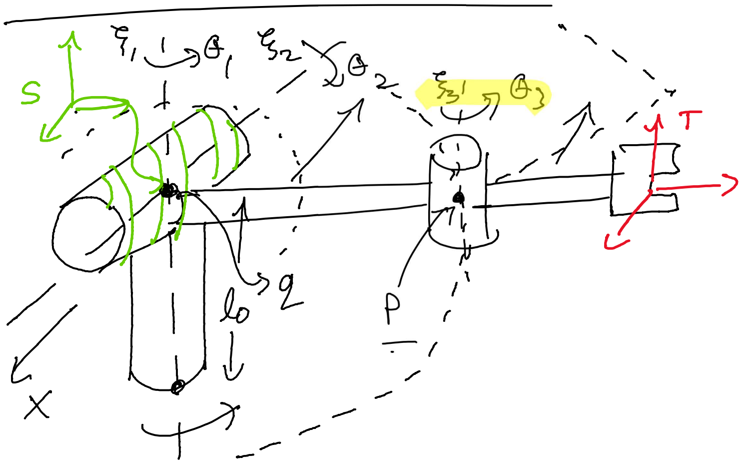
$h\theta_3 + l_1$

$$h\theta_3 + l_1 = \|g p_w - p_0\|$$

$$\boxed{\theta_3 = \frac{\|g p_w - p_0\| - l_1}{h}}$$

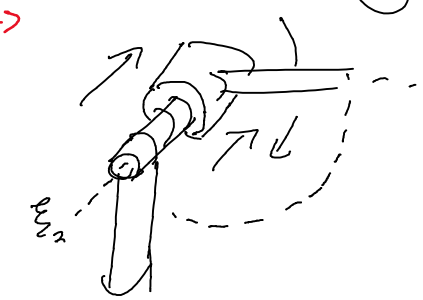
SINGLE value for θ_3. 1 soln.

Screw Joint Problem 2:



Screw Joint: Joint 2

⇒ Often (h)



SCREW MOTION FIRST!!

P.O.E. $$e^{\hat\xi_1\theta_1} e^{\hat\xi_2\theta_2} e^{\hat\xi_3\theta_3} = g_d\, g_{st}^{-1}(0) = g$$

$$e^{\hat\xi_1\theta_1} e^{\hat\xi_2\theta_2} p = gp$$

"Pattern 1"
⇒ transf. on the front ⇒ Subtr. a pt, take the norm

$$e^{\hat\xi_1\theta_1} e^{\hat\xi_2\theta_2} p - e^{\hat\xi_1\theta_1} q = gp - q$$

"Pattern 2"
⇒ transf. towards the back ⇒ Apply a pt. on axis to KO

$$\| e^{\hat\xi_1\theta_1} (e^{\hat\xi_2\theta_2} p - q) \| = \| gp - q \|$$

$$\| e^{\hat\xi_2\theta_2} p - q \| = \| gp - q \|$$

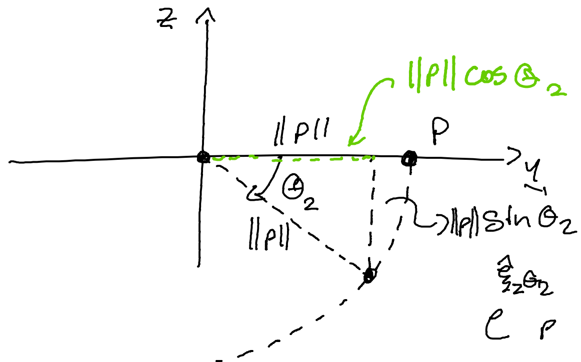
⇒ knocked down to ONE norm w/ 1 transf!

$e^{\hat{\xi}_2\theta_2} p \Rightarrow$ What will this do to p?

$\Rightarrow$ Rotates + translates point p in space!

$$\left\| e^{\hat{\xi}_2\theta_2} p - \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\| = \|gp - q\|$$

$$\| e^{\hat{\xi}_2\theta_2} p \| = \|gp\|$$



z-coord: $-\|p\| \sin\theta_2$

y-coord: $\|p\| \cos\theta_2$

$$e^{\hat{\xi}_2\theta_2} p = \begin{bmatrix} x?? \\ y\text{coord} \\ z\text{coord} \end{bmatrix}$$

Amount of travel: $h\theta_2$ (back into page)

$$\| e^{\hat{\xi}_2\theta_2} p \| = \left\| \begin{bmatrix} -h\theta_2 \\ \|p\| \cos\theta_2 \\ -\|p\| \sin\theta_2 \end{bmatrix} \right\|$$

$$\| e^{\hat{\xi}_2\theta_2} p \|^2 = \|gp\|^2$$

$$(-h\theta_2)^2 + (\|p\| \cos\theta_2)^2 + (-\|p\| \sin\theta_2)^2 = \|gp\|^2$$

$$h^2\theta_2^2 + \|p\|^2 \cos^2\theta_2 + \|p\|^2 \sin^2\theta_2 = \|gp\|^2$$

$$h^2\theta_2^2 + \|p\|^2 (\underline{\cos^2\theta_2 + \sin^2\theta_2}) = \|gp\|^2$$

$$h^2\theta_2^2 + \|p\|^2 = \|gp\|^2$$

$$\theta_2 = \pm\sqrt{\frac{\|gp\|^2 - \|p\|^2}{h^2}}$$