LOST 1012

Wednesday, October 12, 2022 5:15 PM

LOST SECTION 10/12

TODAY'S AGENDA:

1. Image Formation
2. Two-view reconstruction + demo

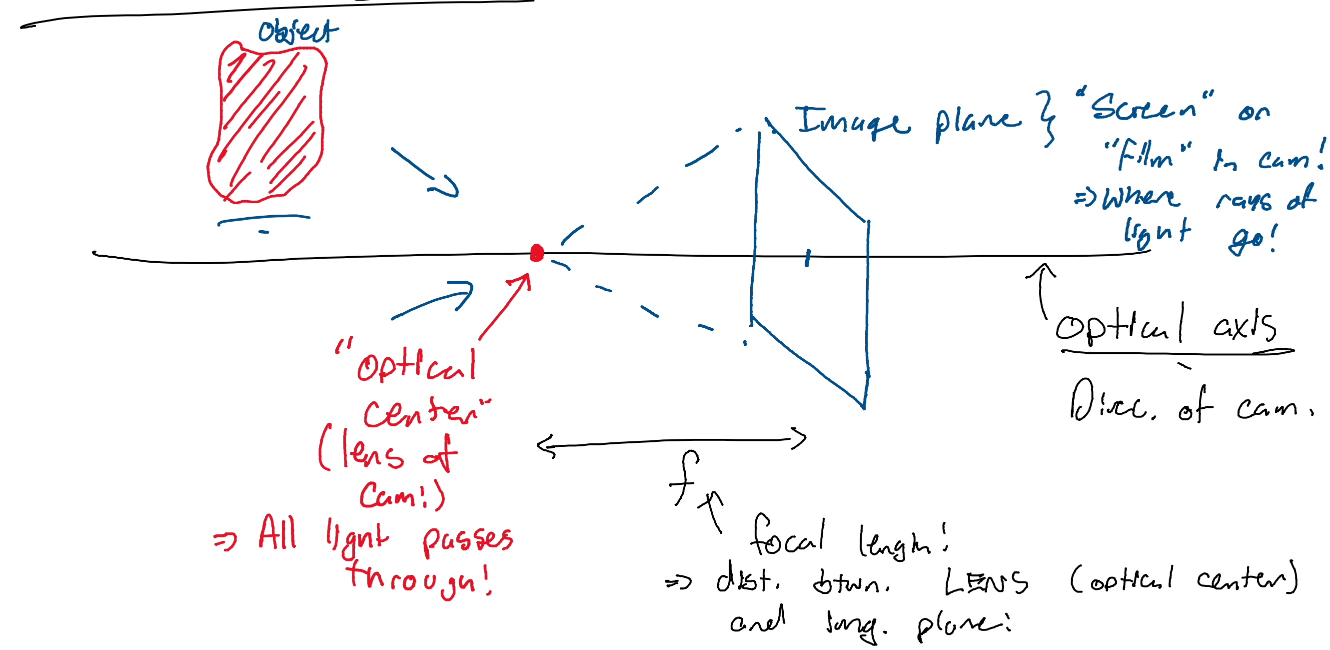
Image Formation:

Q: how are images formed?

⇒ Come up w/ a CAM MODEL!

- Complexity: Complex models for cameras are impractical!
  - ⇒ want to find something w/ a BALANCE of simplicity + accuracy!

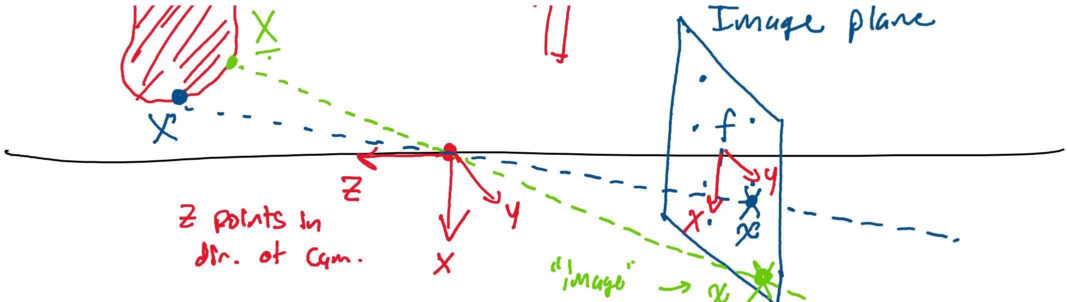
PINHOLE CAMERA MODEL:



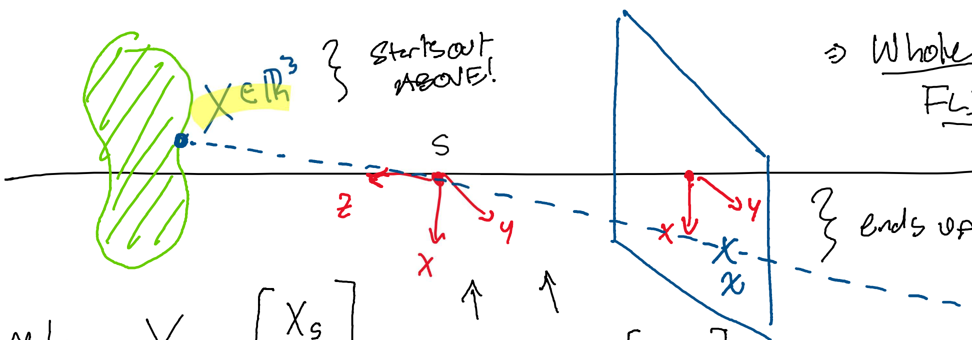
- How does light interact w/ img. plane?

Math. model?

⇒ Relate X → x ?



⇒ Intersections of RAYS w/ img. plane give us the IMAGES!



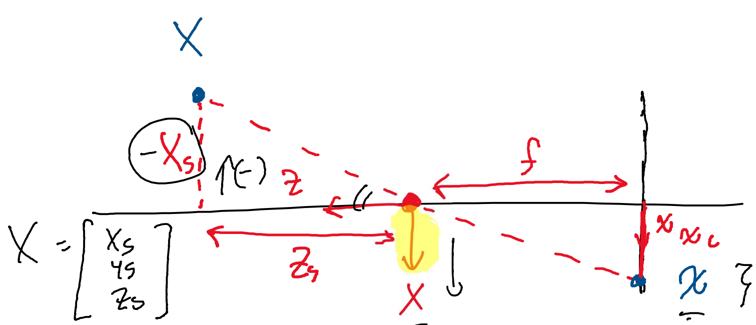
⇒ Whole img. gets FLIPPED b/c of (-) sign!!

rel: $X = \begin{bmatrix} X_s \\ Y_s \\ Z_s \end{bmatrix} \longleftrightarrow x = \begin{bmatrix} x_c \\ y_c \end{bmatrix}$

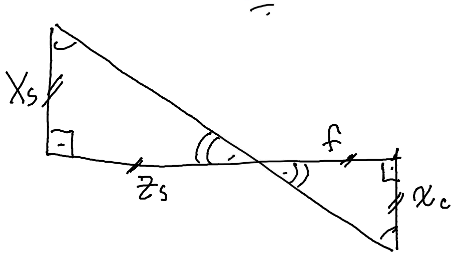
S for spatial

C for cam!

Try looking for a GEOM. rel!



$x = \begin{bmatrix} x_c \\ y_c \end{bmatrix}$



These triangles are "SIMILAR"

- They just differ by a SCALE!

$$\frac{Z_s}{f} = \frac{-X_s}{x_c}$$

Solve for x_c

$$Z_s x_c = -f X_s$$

$$\boxed{x_c = \frac{-f X_s}{Z_s}}$$

$$\boxed{y_c = \frac{-f Y_s}{Z_s}}$$

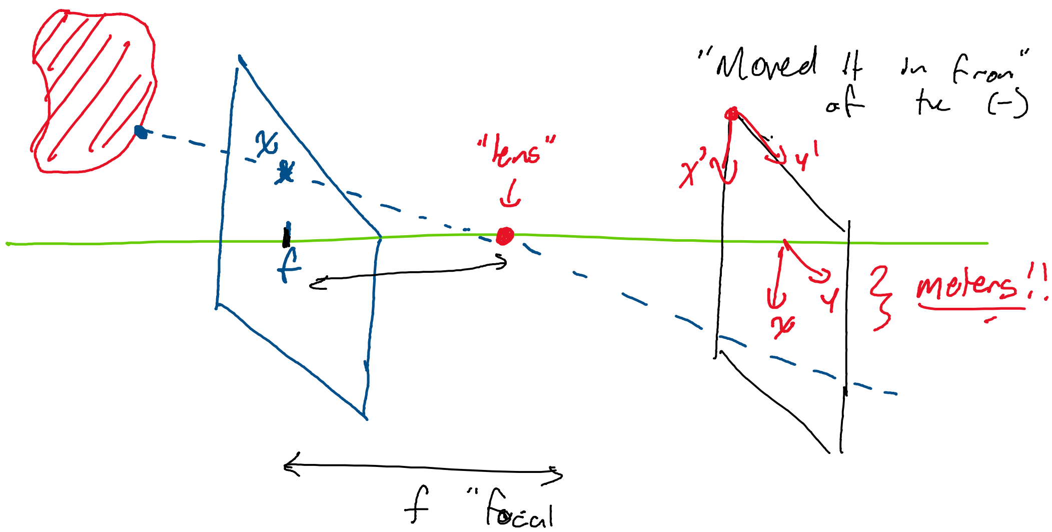
} pixel coord. ENTIRELY in terms of WORLD coords!

⇒ Pixel locations!!

⇒ If these (−) signs are a problem, let's just drop them!!

$$x_c = \frac{f X_s}{Z_s} \qquad y_c = \frac{f Y_s}{Z_s}$$

"Moved it in front" to "take care" of the (−) sign!!



"How can we add matrices"

$$X \rightarrow x \quad \text{} \} \text{ in matrix form!}$$

⇒ Rewrite in HOMOGENEOUS coords!

homog. ↘

$$\bar{X} = \begin{bmatrix} X_s \\ Y_s \\ Z_s \\ 1 \end{bmatrix} \in \mathbb{R}^4 \qquad \bar{x} = \begin{bmatrix} x_c \\ y_c \\ 1 \end{bmatrix} \in \mathbb{R}^3$$

- BREAK our prob. into parts!

1. Reshape $\bar{X}$ to the size of $\bar{x}$
2. Change 3D homog. vec. into IMG. coords!

(1) Reshape $\bar{X}$:

$$\underset{3\times1}{\begin{bmatrix} X_s \\ Y_s \\ Z_s \end{bmatrix}} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{\Pi_0} \underset{4\times1}{\begin{bmatrix} X_s \\ Y_s \\ Z_s \\ 1 \end{bmatrix}}$$

X_s, Y_s, Z_s } Stay the same!

1 } go to zero!

Π_0 "pi = projection"

(2) SCALE into our $\bar{x}$ img. coords!

$$Z_s \begin{bmatrix} x_c \\ y_c \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_s \\ Y_s \\ Z_s \end{bmatrix}$$

$$x_c = \frac{f X_s}{Z_s} \qquad y_c = \frac{f Y_s}{Z_s}$$

Very clean matrix by mult. by Z_s

$$Z_s x_c = f X_s$$

$$Z_s y_c = f Y_s$$

" K_f "

=> focal calibration matrix!

Spatial $\bar{X}$

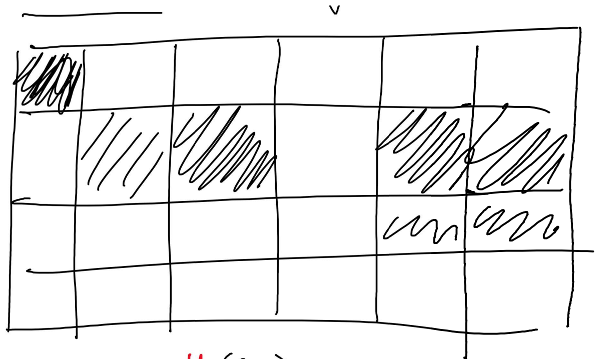
$$Z_s \begin{bmatrix} x_c \\ y_c \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X_s \\ Y_s \\ Z_s \\ 1 \end{bmatrix}$$

$$\boxed{Z_s \bar{x} = K_f \Pi_0 \bar{X}}$$

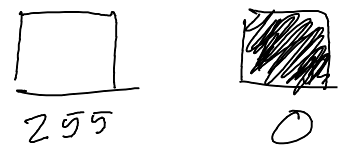
☆ one repr. of our pinhole cam! ☆

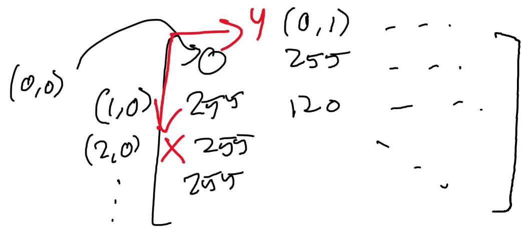
$$= K_f \bar{X} \rightarrow \begin{bmatrix} X_s \\ Y_s \\ Z_s \end{bmatrix}$$

- let's discuss DIGITAL images!



⇒ Img: MATRIX.
"Intensity" values!





X & Y are your row & col!
⇒ our PIXEL coords!

1. Change the ORIGIN

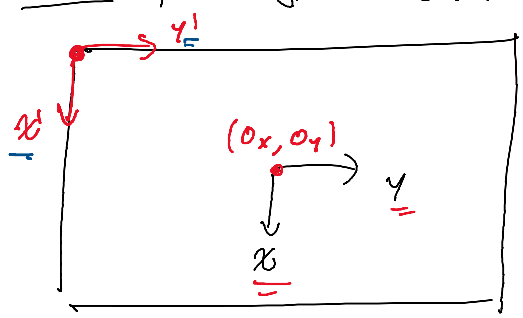
2. Scale our coords. into PIXELS (from m, mm, ...)

$$K_s = \begin{bmatrix} S_x & S_\theta & O_x \\ 0 & S_y & O_y \\ 0 & 0 & 0 \end{bmatrix}$$

↓ Scales m → pixels

→ MOVE the origin of our image!

"Scaling Calibration matrix"



Complete transf. into PIXEL coords:

$$z_s \tilde{x} = K_s K_f \Pi_0 \bar{X}$$

→ spatial points!

Scale, focal, reshape (project)

$K = K_s K_f$ "CAMERA CALIBRATION MATRIX"

480 x 720 img
$O_x = 240$
$O_y = 360$

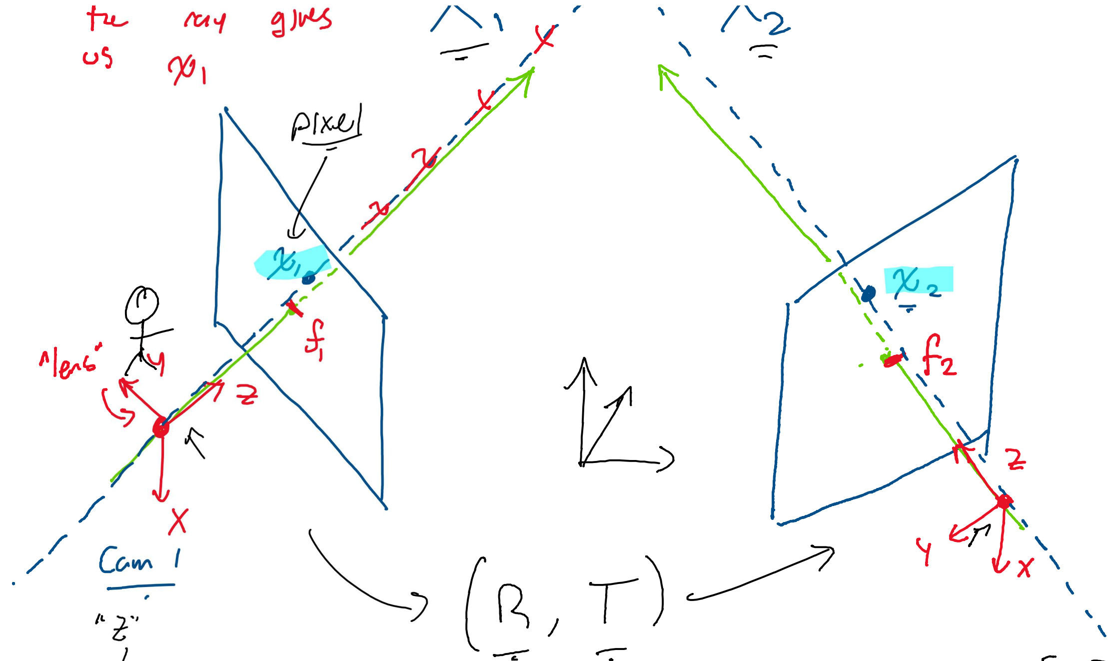
TWO VIEW RECONSTR:

⇒ RECOVER DEPTH FROM IMAGES!
⇒ Important for robotics! ex: obstacles!

⇒ Epipolar constr + Essential matrix!

Part 1: Recovering Depth:

⇒ Everything along



What we know: x_1, x_2

What we want → either X_1 or X

"Z"

$$\lambda_1 \bar{x}_1 = K_1 \Pi_0 X_1$$

$$\boxed{\lambda_1 \bar{x}_1 = K_1 X_1}$$

$$\boxed{\lambda_2 \bar{x}_2 = K_2 X_2}$$

$$\begin{bmatrix} 1 & & & 0 \\ & 1 & & 0 \\ & & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

sets "1" in homog. coords to 0!

$$\begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix}$$

$$\lambda_1 \bar{x}_1 = K_1 X_1$$

$$\lambda_1 K_1^{-1} \bar{x}_1 = X_1$$

"Normalized coords"

$$x_1 = K_1^{-1} \bar{x}_1$$

$$\left\{ \begin{aligned} \lambda_1 x_1 &= X_1 \\ \lambda_2 x_2 &= X_2 \end{aligned} \right\}$$

$$X_1 = \begin{bmatrix} X_s \\ Y_s \\ Z_s \end{bmatrix}$$

How can we RELATE x_1 & x_2?

$$X_2 = R X_1 + T$$

$$\lambda_2 x_2 = R(\lambda_1 x_1) + T$$

} ONLY things we DON'T: λ_1, λ_2

$$\lambda_2 x_2 = \lambda_1 R x_1 + T$$

$$-\lambda_1 R x_1 + \lambda_2 x_2 = T$$

$$v = \begin{bmatrix} \lambda_1 \\ \vdots \end{bmatrix}$$

$[\lambda_2]$

$$\underset{\substack{\uparrow \\ \text{matrix!} \\ \text{(known!)}}}{A}\ \underset{\substack{\uparrow \\ \text{vector of} \\ \text{unknowns!}}}{v} = \underset{\substack{\uparrow \\ \text{known!}}}{b} \qquad \underline{v = A^{-1}b}$$

$$\underbrace{\begin{bmatrix} | & | \\ -Rx_1 & x_2 \\ | & | \end{bmatrix}}_{\text{"A"}} \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = T$$

3x1 vec! 3x1 vec

} Can solve for λ_1 & λ_2!

$A \in \mathbb{R}^{3\times 2}$ matrix!

"Pseudo Inverse" ⇒ "Moore-Penrose pseudoinv"

$$A^+ = (A^TA)^{-1}A^T$$ } Pseudoinv. of A

$$A^+A = (A^TA)^{-1}\underline{A^T \cdot A} = (A^TA)^{-1}(A^TA) = I$$

$$\cancel{\begin{bmatrix} | & | \\ -Rx_1 & x_2 \\ | & | \end{bmatrix}^+ \begin{bmatrix} | & | \\ -Rx_1 & x_2 \\ | & | \end{bmatrix}} \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} | & | \\ -Rx_1 & x_2 \\ | & | \end{bmatrix}^+ T$$

$$\boxed{\begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} | & | \\ -Rx_1 & x_2 \\ | & | \end{bmatrix}^+ T}$$

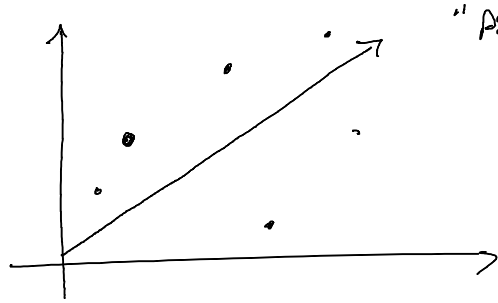
⇒ gives us our depths!!

$\lambda_1 x_1 = X_1$

$\lambda_2 x_2 = X_2$

} Both point to same physical pt.

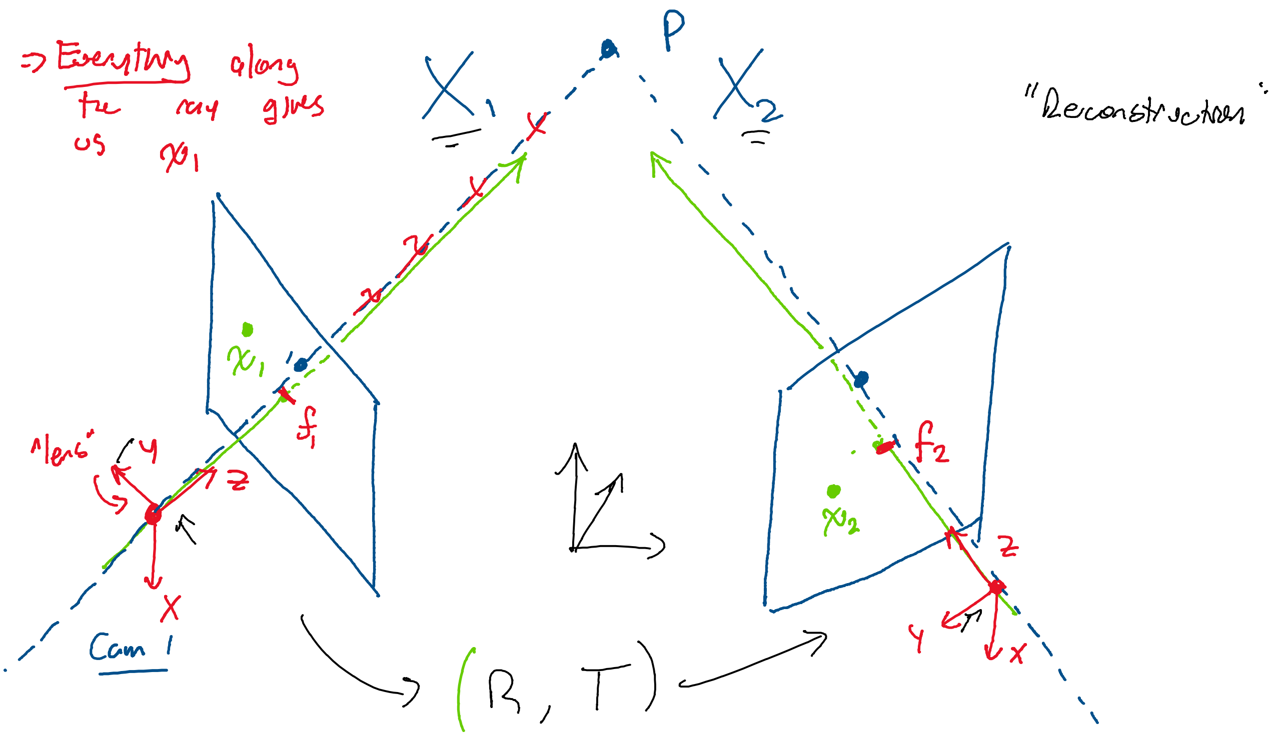
"Least sq. soln"



"pseudoinv" gives BEST soln. for that case.

ESSENTIAL MATRIX

ESSENTIAL MATRIX:



How do we know if x_1, x_2 (pixels) POINT to the same 3D SPATIAL location??

(from two images!)

$$X_2 = R X_1 + T$$

$$\lambda_2 x_2 = \lambda_1 R x_1 + T$$

⇒ REDUCE to the SIMPLEST constr. possible!
- elim. unknowns! (λ_1, λ_2)
- eliminate unnec. terms.

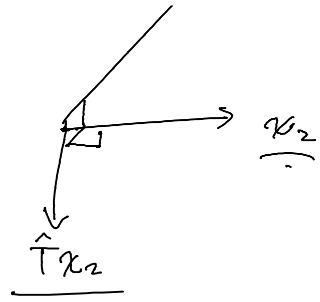
$$T \times V = \underbrace{\hat{T}}_{\text{matrix!!}} V$$

$$T \times T = 0$$

$$\lambda_2 \hat{T} x_2 = \lambda_1 \hat{T} R x_1 + \cancel{\hat{T} T}^{\,0}$$

$$\lambda_2 \hat{T} x_2 = \lambda_1 \hat{T} R x_1$$

x^T $\quad X \perp y \quad X \cdot y = x^T y$



$X \cdot Y = 0$

$X \cdot Y = \|X\| \|Y\| \cos\theta$
$= \|X\| \|Y\| \cos(90°)$
$= 0$

$$\lambda_2 x_2^T \hat{T} x_2 = \lambda_1 x_2^T \hat{T} R x_1$$

$$0 = \lambda_1 x_2^T \hat{T} R x_1$$

$$\boxed{0 = x_2^T \hat{T} R x_1}$$ "EPIPOLAR CONSTR!"

⇒ If x_2 & x_1 corr. to the SAME pt. in space:

$$x_2^T \hat{T} R x_1 = 0$$

⇒ "ESSENTIAL MATRIX" $\boxed{E = \hat{T} R}$

$$x_2^T E x_1 = 0$$

If x_2 & x_1 are from the same physical location.

$X_1, X_2, \ldots, X_8$

$$\chi E^s = 0$$

χ: Matrix that encodes information about $X_1, \ldots, X_8$

9×9 "Kronecker product" $\otimes$

$E^s = ??$ $E^s = \begin{bmatrix} e_{11} \\ e_{21} \\ e_{31} \\ \vdots \\ \vdots \\ e_{33} \end{bmatrix} \in \mathbb{R}^9$ $E = \begin{bmatrix} e_{11} & \cdots & \\ e_{21} & \ddots & \\ \vdots & & e_{33} \end{bmatrix}$

don't know, want to recover!!

$$\chi E^s = 0$$

⇒ E^s should be UNIQUE up to a scale factor!!

⇒ If E^s ONLY points in a single dir, χ is a 9x9 matrix:

⇒ Null space of χ to be 1D

⇒ Column space of χ is 8D

- MUST PICK 8 pts. to get χ a 1D null space, which gives E^s ⇒ E up to a scale factor.

$$\lambda_{..} T_{..} + \lambda_{..} T + \lambda_{..} T = 0$$

$$\begin{bmatrix} | & | & \\ ..T & ..T & ..T \\ | & | & \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} = 0$$

$$\text{pt cloud} = \begin{bmatrix} | & & | & & | \\ X_1 & \cdots & X_j & \cdots & X_n \\ | & & | & & | \end{bmatrix}$$

$$\|\text{ptcloud}\| = [\|x_1\| \; \cdots \; \cancel{\|x_j\|} \; \cdots \; \|x_n\|]$$

np. argmin$(\| \cdots \|)$

$$U_{nn} = \begin{bmatrix} X_j & X_{j+10} \end{bmatrix}$$