Last Time

Chapter 2 Rigid Body Motion

2 Rotational motion in $\mathbb{R}^3$

- *Quaternions*

3 Rigid Motion in $\mathbb{R}^3$

- SE(3)
  - As a Configuration Space
  - *Homogeneous Representation*
  - *SE(3) is a Group*
  - *SE(3) is a Rigid Body Transformation*

Recap

Today

Chapter 2 Rigid Body Motion

3 Rigid Motion in $\mathbb{R}^3$

- *Exponential coordinates of SE(3)*
  - *Twists*
  - *se(3)*
  - *The Exponential Map*
- *Screw Motion*
  - *What is a Screw*
  - *Twist associated with a Screw*
  - *Screw associated with a Twist*

Chapter 3 Manipulator Kinematics

1 Forward kinematics

- *Joint Space*

2.3 Rigid motion in $\mathbb{R}^3$

Exponential coordinates of $SE(3)$:

For rotational motion:

$$\dot{p}(t)=\omega \times (p(t) - q)$$

$$\begin{bmatrix} \dot{p} \\ 0 \end{bmatrix}=\begin{bmatrix} \hat{\omega} & -\omega \times q \\ 0 & 0 \end{bmatrix}\begin{bmatrix} p \\ 1 \end{bmatrix}$$

$$\text{or } \dot{\bar{p}}=\hat{\xi} \cdot \bar{p} \Rightarrow \bar{p}(t) = e^{\hat{\xi}t}\bar{p}(0)$$

$$\text{where } e^{\hat{\xi}t}=I + \hat{\xi}t + \frac{(\hat{\xi}t)^2}{2!} + \cdots$$

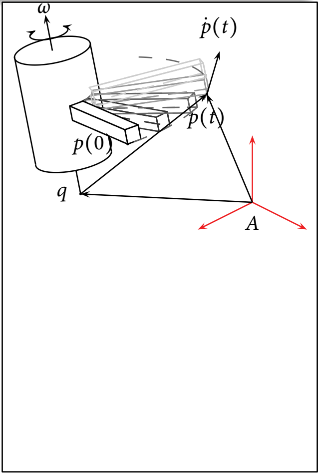

Figure 2.13

2.3 Rigid motion in $\mathbb{R}^3$

Exponential coordinates of $SE(3)$:

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For translational motion:

$$\dot{p}(t)=v$$

$$\begin{bmatrix} \dot{p}(t) \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & v \\ 0 & 0 \end{bmatrix} \begin{bmatrix} p \\ 1 \end{bmatrix}$$

$$\dot{\bar{p}}(t)=\hat{\xi} \cdot \bar{p}(t) \Rightarrow \bar{p}(t) = e^{\hat{\xi} t}\bar{p}(0)$$

$$\hat{\xi}=\begin{bmatrix} 0 & v \\ 0 & 0 \end{bmatrix}$$

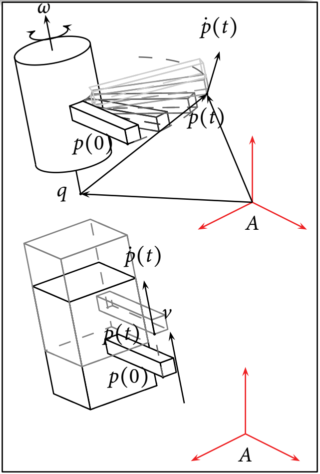

Figure 2.13

Today

Chapter 2 Rigid Body Motion

3 Rigid Motion in $\mathbb{R}^3$

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Chapter 3 Manipulator Kinematics

1 Forward kinematics

- *Joint Space*

2.3 Rigid motion in $\mathbb{R}^3$

Definition:

$$se(3) = \left\{ \begin{bmatrix} \hat{\omega} & v \\ 0 & 0 \end{bmatrix} \in \mathbb{R}^{4\times4} \middle| v, \omega \in \mathbb{R}^3 \right\}$$

is called the twist space. There exists a 1-1 correspondence between $se(3)$ and $\mathbb{R}^6$, defined by $\wedge : \mathbb{R}^6 \mapsto se(3)$

$$\xi := \begin{bmatrix} v \\ \omega \end{bmatrix} \mapsto \hat{\xi} = \begin{bmatrix} \hat{\omega} & v \\ 0 & 0 \end{bmatrix}$$

Today

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Property 6: $\exp : se(3) \mapsto SE(3), \hat{\xi}\theta \mapsto e^{\hat{\xi}\theta}$

2.3 Rigid motion in $\mathbb{R}^3$

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Property 6: $\exp : se(3) \mapsto SE(3), \hat{\xi}\theta \mapsto e^{\hat{\xi}\theta}$

Proof :

Let $\hat{\xi} = \begin{bmatrix} \hat{\omega} & v \\ 0 & 0 \end{bmatrix}$

- If $\omega = 0$, then $\hat{\xi}^2 = \hat{\xi}^3 = \cdots = 0$, $e^{\hat{\xi}\theta} = \begin{bmatrix} I & v\theta \\ 0 & 1 \end{bmatrix} \in SE(3)$

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2.3 Rigid motion in $\mathbb{R}^3$

- If ω is not 0, assume $\|\omega\| = 1$.

Define:

$$g_0 = \begin{bmatrix} I & \omega \times v \\ 0 & 1 \end{bmatrix}, \hat{\xi}' = g_0^{-1} \cdot \hat{\xi} \cdot g_0 = \begin{bmatrix} \hat{\omega} & h\omega \\ 0 & 0 \end{bmatrix}$$

where $h = \omega^T \cdot v$.

$$e^{\hat{\xi}\theta} = e^{g_0 \cdot \hat{\xi}' \cdot g_0^{-1}} = g_0 \cdot e^{\hat{\xi}'\theta} \cdot g_0^{-1}$$

and as

$$\hat{\xi}'^2 = \begin{bmatrix} \hat{\omega}^2 & 0 \\ 0 & 0 \end{bmatrix}, \hat{\xi}'^3 = \begin{bmatrix} \hat{\omega}^3 & 0 \\ 0 & 0 \end{bmatrix}$$

we have

$$e^{\hat{\xi}'\theta} = \begin{bmatrix} e^{\hat{\omega}\theta} & h\omega\theta \\ 0 & 1 \end{bmatrix} \Rightarrow e^{\hat{\xi}\theta} = \begin{bmatrix} e^{\hat{\omega}\theta} & (I - e^{\hat{\omega}\theta})\hat{\omega}v + \omega\omega^T v\theta \\ 0 & 1 \end{bmatrix}$$

□

2.3 Rigid motion in $\mathbb{R}^3$

$$p(\theta) = e^{\hat{\xi}\theta} \cdot p(0) \Rightarrow g_{ab}(\theta) = e^{\hat{\xi}\theta}$$

If there is offset,

$$g_{ab}(\theta) = e^{\hat{\xi}\theta} g_{ab}(0) (\text{ Why?})$$

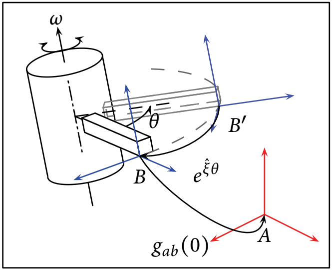

Figure 2.14

2.3 Rigid motion in $\mathbb{R}^3$

Property 7: $exp : se(3) \mapsto SE(3)$ is onto.

2.3 Rigid motion in $\mathbb{R}^3$

Property 7: $exp : se(3) \mapsto SE(3)$ is onto.

Proof :

Let $g = (p, R), R \in SO(3), p \in \mathbb{R}^3$

Case 1: $(R = I)$ Let

$$\hat{\xi} = \begin{bmatrix} 0 & \frac{p}{\|p\|} \\ 0 & 0 \end{bmatrix}, \theta = \|p\| \Rightarrow e^{\hat{\xi}\theta} = g = \begin{bmatrix} I & p \\ 0 & 1 \end{bmatrix}$$

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Chapter 2 Rigid Body Motion

3 Rigid Motion in $\mathbb{R}^3$

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Chapter 3 Manipulator Kinematics

1 Forward kinematics

- *Joint Space*

2.3 Rigid motion in $\mathbb{R}^3$

□ Screws, twists and screw motion:

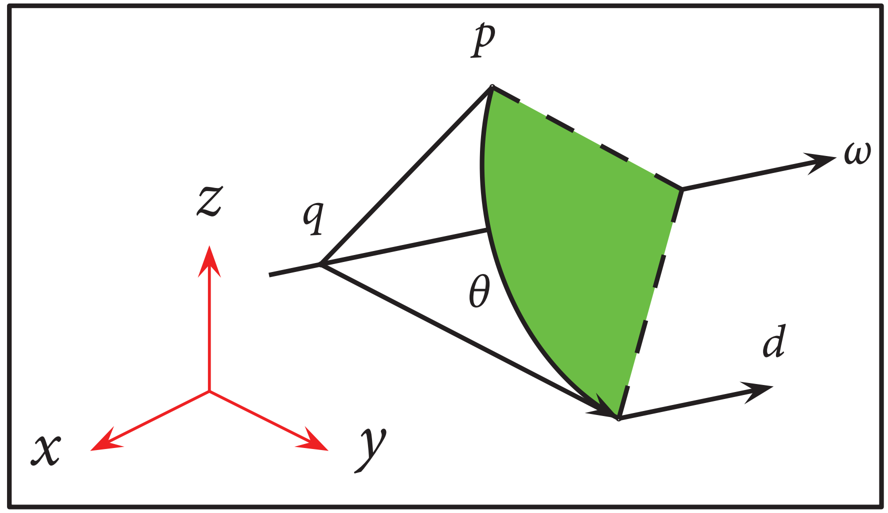

Figure 2.15

Screw attributes

Pitch: $h = \frac{d}{\theta}(\theta = 0, h = \infty), d = h \cdot \theta$

Axis: $l = \{q + \lambda\omega | \lambda \in \mathbb{R}\}$

Magnitude: $M = \theta$

2.3 Rigid motion in $\mathbb{R}^3$

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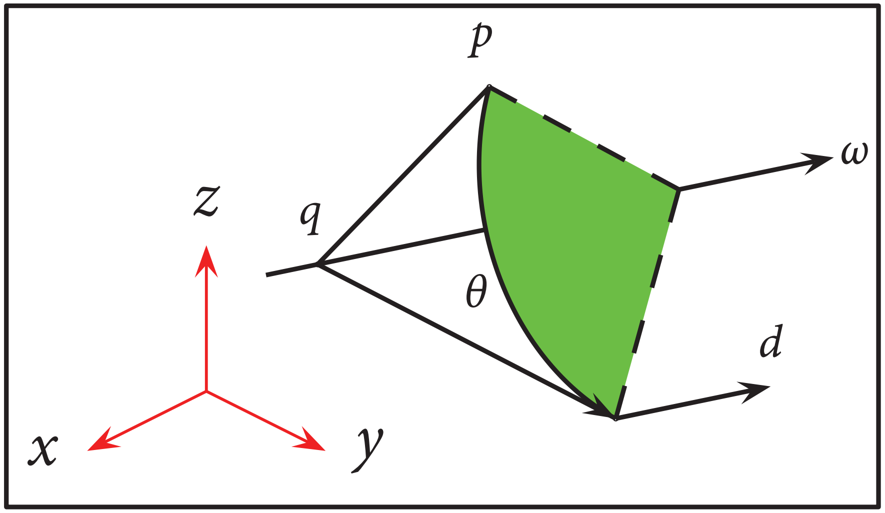

Figure 2.15

Screw attributes

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Axis: $l = \{q + \lambda\omega | \lambda \in \mathbb{R}\}$

Magnitude: $M = \theta$

Definition:
A **screw** S consists of an axis l, pitch h, and magnitude M. A **screw motion** is a rotation by $\theta = M$ about l, followed by translation by $h\theta$, parallel to l. If $h = \infty$, then, translation about v by $\theta = M$

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2.3 Rigid motion in $\mathbb{R}^3$

Corresponding $g \in SE(3)$:

$$g \cdot p = q + e^{\hat{\omega}\theta}(p - q) + h\theta\omega$$

$$g \cdot \begin{bmatrix} p \\ 1 \end{bmatrix} = \begin{bmatrix} e^{\hat{\omega}\theta} & (I - e^{\hat{\omega}\theta})q + h\theta\omega \\ 0 & 1 \end{bmatrix} \begin{bmatrix} p \\ 1 \end{bmatrix} \Rightarrow$$

$$g = \begin{bmatrix} e^{\hat{\omega}\theta} & (I - e^{\hat{\omega}\theta})q + h\theta\omega \\ 0 & 1 \end{bmatrix}$$

2.3 Rigid motion in $\mathbb{R}^3$

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$$g = \begin{bmatrix} e^{\hat{\omega}\theta} & (I - e^{\hat{\omega}\theta})q + h\theta\omega \\ 0 & 1 \end{bmatrix}$$

On the other hand...

$$e^{\hat{\xi}\theta} = \begin{bmatrix} e^{\hat{\omega}\theta} & (I - e^{\hat{\omega}\theta})\omega \times v + \omega\omega^T v\theta \\ 0 & 1 \end{bmatrix}$$

If we let $v = -\omega \times q + h\omega$, then

$$(I - e^{\hat{\omega}\theta})(-\hat{\omega}^2 q) = (I - e^{\hat{\omega}\theta})(-\omega\omega^T q + q) = (I - e^{\hat{\omega}\theta})q$$

Thus, $e^{\hat{\xi}\theta} = g$

2.3 Rigid motion in $\mathbb{R}^3$

Corresponding $g \in SE(3)$:

$$g \cdot p = q + e^{\hat{\omega}\theta}(p - q) + h\theta\omega$$

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$$(I - e^{\hat{\omega}\theta})(-\hat{\omega}^2 q) = (I - e^{\hat{\omega}\theta})(-\omega\omega^T q + q) = (I - e^{\hat{\omega}\theta})q$$

Thus, $e^{\hat{\xi}\theta} = g$

For pure rotation $(h = 0)$: $\xi = (-\omega \times q, \omega)$

For pure translation: $g = \begin{bmatrix} I & v\theta \\ 0 & 1 \end{bmatrix}$, $\Rightarrow \xi = (v, 0)$, and $e^{\hat{\xi}\theta} = g$

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Chapter 3 Manipulator Kinematics

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2.3 Rigid motion in $\mathbb{R}^3$

□ Screw associated with a twist:

$\xi = (v, \omega) \in \mathbb{R}^6$

1. Pitch: $h = \begin{cases} \dfrac{\omega^T v}{\|\omega\|^2}, & \text{if } \omega \neq 0 \\ \infty, & \text{if } \omega = 0 \end{cases}$
2. Axis: $l = \begin{cases} \dfrac{\omega \times v}{\|\omega\|^2} + \lambda\omega, & \lambda \in \mathbb{R}, \text{if } \omega \neq 0 \\ 0 + \lambda v & \lambda \in \mathbb{R}, \text{if } \omega = 0 \end{cases}$
3. Magnitude: $M = \begin{cases} \|\omega\|, & \text{if } \omega \neq 0 \\ \|v\|, & \text{if } \omega = 0 \end{cases}$

Screw associated with a twist:

$\xi = (v, \omega) \in \mathbb{R}^6$

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Special cases:

1. $h = \infty$, Pure translation (prismatic joint)
2. $h = 0$, Pure rotation (revolute joint)

2.3 Rigid motion in $\mathbb{R}^3$

Screw	**Twist: $\hat{\xi}\theta$**
Case 1: Pitch: $h = \infty$ Axis: $l = \{q + \lambda v \mid \|v\| = 1, \lambda \in \mathbb{R}\}$ Magnitude: M	$\theta = M,$ $\hat{\xi} = \begin{bmatrix} 0 & v \\ 0 & 0 \end{bmatrix}$
Case 2: Pitch: $h \neq \infty$ Axis: $l = \{q + \lambda \omega \mid \|\omega\| = 1, \lambda \in \mathbb{R}\}$ Magnitude: M	$\theta = M,$ $\hat{\xi} = \begin{bmatrix} \hat{\omega} & -\hat{\omega}q + h\omega \\ 0 & 0 \end{bmatrix}$

2.3 Rigid motion in $\mathbb{R}^3$

Screw	Twist: $\hat{\xi}\theta$
Case 1: Pitch: $h = \infty$ Axis: $l = \{q + \lambda v \mid \|v\| = 1, \lambda \in \mathbb{R}\}$ Magnitude: M	$\theta = M,$ $\hat{\xi} = \begin{bmatrix} 0 & v \\ 0 & 0 \end{bmatrix}$
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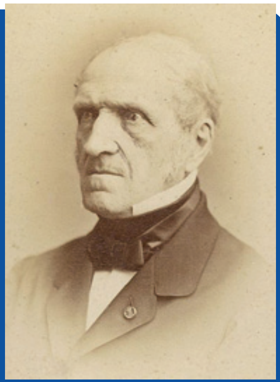
Definition: Screw Motion

Rotation about an axis by $\theta = M$, followed by translation about the same axis by $h\theta$

2.3 Rigid motion in $\mathbb{R}^3$

Theorem 2 (Chasles):
Every rigid body motion can be realized by a rotation about an axis combined with a translation parallel to that axis.



1793–1880

2.3 Rigid motion in $\mathbb{R}^3$

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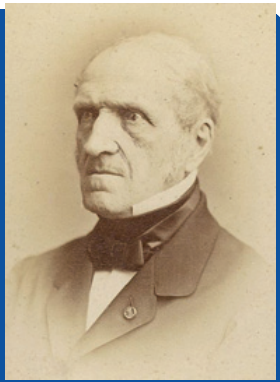

1793–1880

Proof :
For $\hat{\xi} \in se(3)$:

$$\hat{\xi} = \hat{\xi}_1 + \hat{\xi}_2 = \begin{bmatrix} \hat{\omega} & -\omega \times q \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & h\omega \\ 0 & 0 \end{bmatrix}$$

$$[\hat{\xi}_1, \hat{\xi}_2] = 0 \Rightarrow e^{\hat{\xi}\theta} = e^{\hat{\xi}_1\theta} e^{\hat{\xi}_2\theta}$$

□

† End of Section †

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Chapter 3 Manipulator Kinematics

1 Forward kinematics

- *Joint Space*

Chapter 3 Manipulator Kinematics

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Chapter 3 Manipulator Kinematics

1 Forward kinematics

- *Joint Space*

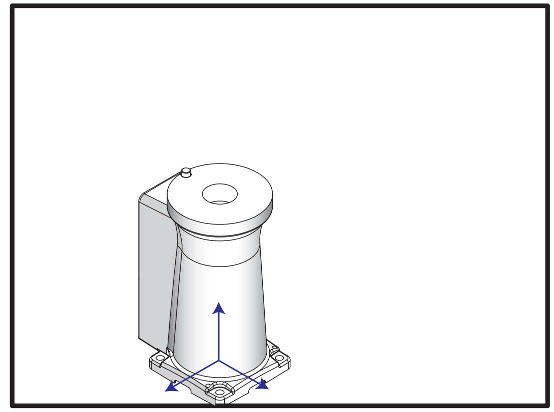
3.1 Forward kinematics

(a) Adept Cobra i600 (SCARA)

Figure 3.1

3.1 Forward kinematics

(a) Adept Cobra i600 (SCARA)

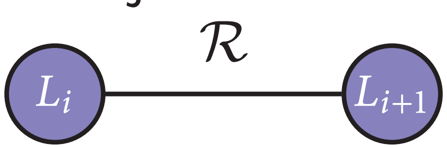


(b) Forward kinematics of SCARA

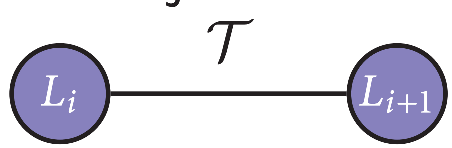
Figure 3.1

◊ Lower Pair Joints:

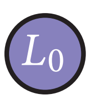
revolute joint $S^1 \mapsto SO(2)$



prismatic joint $\mathbb{R} \mapsto T(1)$



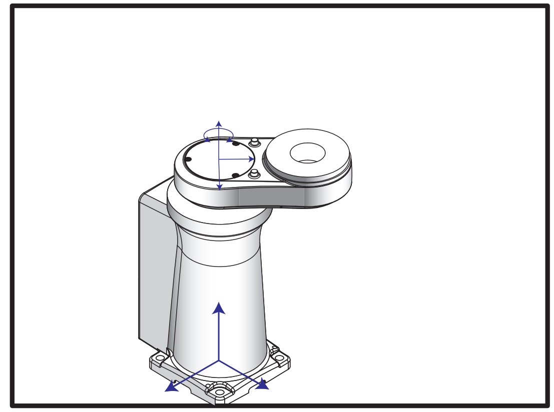
◊ Forward kinematics:



base: stationary

3.1 Forward kinematics

(a) Adept Cobra i600 (SCARA)

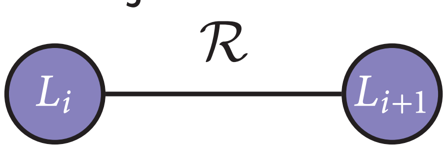


(b) Forward kinematics of SCARA

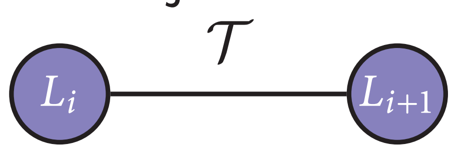
Figure 3.1

◇ Lower Pair Joints:

revolute joint $S^1 \mapsto SO(2)$



prismatic joint $\mathbb{R} \mapsto T(1)$

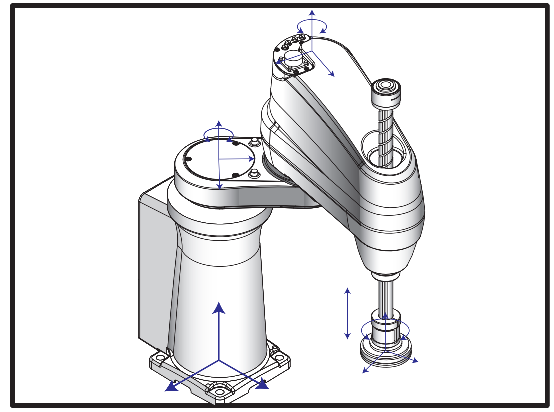


◇ Forward kinematics:

base: stationary Link 1: first movable link

3.1 Forward kinematics

(a) Adept Cobra i600 (SCARA)

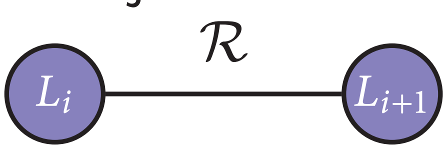


(b) Forward kinematics of SCARA

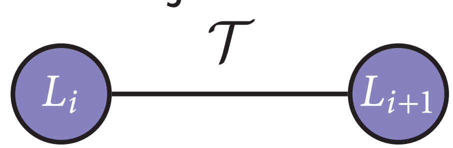
Figure 3.1

◊ Lower Pair Joints:

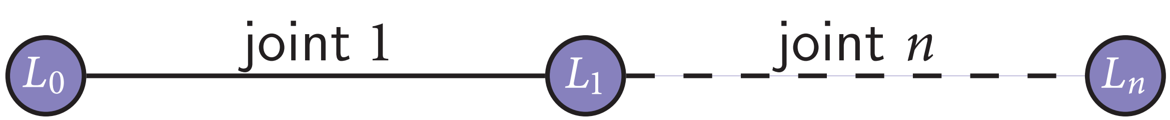
revolute joint $S^1 \mapsto SO(2)$



prismatic joint $\mathbb{R} \mapsto T(1)$



◊ Forward kinematics:



base: stationary   Link 1: first movable link   Link n: end-effector

Robots in Practice

SCARA

3.1 Forward kinematics

Joint space:

Revolute joint: $S^1, \theta_i \in S^1$ or $\theta_i \in [-\pi, \pi]$

Prismatic joint: $\mathbb{R}$

Joint space: $Q : \underbrace{S^1 \times \cdots \times S^1}_{\text{no. of } R \text{ joint}} \times \underbrace{\mathbb{R} \times \cdots \times \mathbb{R}}_{\text{no. of } P \text{ joint}}$

3.1 Forward kinematics

□ Joint space:

Revolute joint: $S^1, \theta_i \in S^1$ or $\theta_i \in [-\pi, \pi]$

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Adept $Q : S^1 \times S^1 \times S^1 \times \mathbb{R}$

Elbow $Q = \Gamma^6 : \underbrace{S^1 \times \cdots \times S^1}_{6}$

3.1 Forward kinematics

□ Joint space:

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Adept $Q : S^1 \times S^1 \times S^1 \times \mathbb{R}$

Elbow $Q = \Gamma^6 : \underbrace{S^1 \times \cdots \times S^1}_{6}$

Reference (nominal) joint config: $\theta = (0, 0, \ldots, 0) \in Q$

Reference (nominal) end-effector config: $g_{st}(0) \in SE(3)$

3.1 Forward kinematics

□ Joint space:

Revolute joint: $S^1, \theta_i \in S^1$ or $\theta_i \in [-\pi, \pi]$

Prismatic joint: $\mathbb{R}$

Joint space: $Q : \underbrace{S^1 \times \cdots \times S^1}_{\text{no. of } R \text{ joint}} \times \underbrace{\mathbb{R} \times \cdots \times \mathbb{R}}_{\text{no. of } P \text{ joint}}$

Adept $\quad Q : S^1 \times S^1 \times S^1 \times \mathbb{R}$

Elbow $\quad Q = \Gamma^6 : \underbrace{S^1 \times \cdots \times S^1}_{6}$

Reference (nominal) joint config: $\theta = (0, 0, \ldots, 0) \in Q$

Reference (nominal) end-effector config: $g_{st}(0) \in SE(3)$

Arbitrary configuration $g_{st}(\theta)$:

$$g_{st} : \theta \in Q \mapsto g_{st}(\theta) \in SE(3)$$

Today

Chapter 2 Rigid Body Motion

3 Rigid Motion in $\mathbb{R}^3$

- *Exponential coordinates of SE(3)*
  - *Twists*
  - *se(3)*
  - *The Exponential Map*
- *Screw Motion*
  - *What is a Screw*
  - *Twist associated with a Screw*
  - *Screw associated with a Twist*

Chapter 3 Manipulator Kinematics

1 Forward kinematics

- *Joint Space*

3.1 Forward kinematics

□ Classical Approach:

$$g_{st}(\theta_1, \theta_2) = g_{st}(\theta_1) \cdot g_{l_1 l_2} \cdot g_{l_2 t}$$

Disadvantage: A coordinate frame needed for each link

3.1 Forward kinematics

□ Classical Approach:

$$g_{st}(\theta_1, \theta_2) = g_{st}(\theta_1) \cdot g_{l_1 l_2} \cdot g_{l_2 t}$$

Disadvantage: A coordinate frame needed for each link

□ The product of exponentials formula:

Consider Fig 3.2.

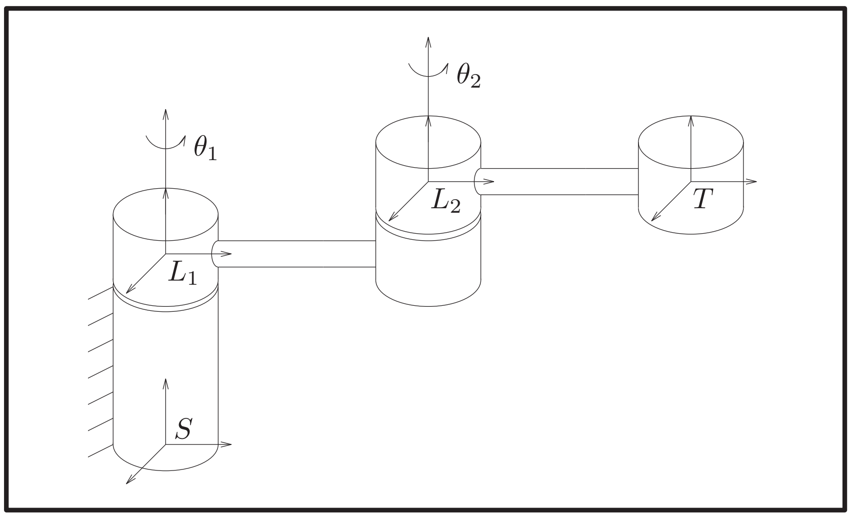

Figure 3.2: A two degree of freedom manipulator

(Continues next slide)