Last Time

Chapter 2 Rigid Body Motion

1 Rigid Body Transformations

- Length Preserving: $||g(p) - g(q)|| = ||p - q||$
- Orientation Preserving: $g_*(v \times w) = g_*(v) \times g_*(w)$

2 Rotational motion in $\mathbb{R}^3$

- Rotation Matrix
  - Represents *configuration*
  - Represents *(rotational) transformation*
- Rotation Matrices with matrix multiplication form a *Group*
- Rotational Transformation is a *Rigid Body Transformation*

Recap

Today

Chapter 2 Rigid Body Motion

2 Rotational motion in $\mathbb{R}^3$

- The *Exponential Map*
- Rodrigues *Formula*
- *Euler Angles*
- *Quaternions*

2.2 Rotational Motion in $\mathbb{R}^3$

□ Parametrization of $SO(3)$ (the exponential coordinate):

◇ **Review:** $S^1 = \{z \in \mathbb{C} \| |z| = 1\}$

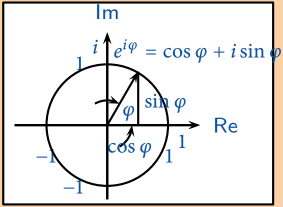

Figure 2.4

Euler's Formula

"One of the most remarkable, almost astounding, formulas in all of mathematics."

R. Feynman

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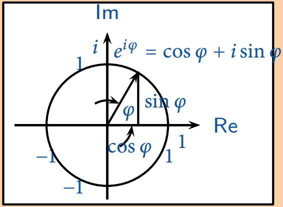

Figure 2.4

Euler's Formula

"One of the most remarkable, almost astounding, formulas in all of mathematics."

R. Feynman

◇ **Review:**

$$\begin{cases} \dot{x}(t) = ax(t) \\ x(0) = x_0 \end{cases} \Rightarrow x(t) = e^{at}x_0$$

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2.2 Rotational Motion in $\mathbb{R}^3$

$$R \in SO(3), R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

$$r_i \cdot r_j = \begin{cases} 0, & i \neq j \\ 1, & i = j \end{cases} \quad \leftarrow 6 \text{ constraints}$$

$\Rightarrow$ 3 independent parameters!

2.2 Rotational Motion in $\mathbb{R}^3$

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Consider motion of a point q on a rotating link

$$\begin{cases} \dot{q}(t) = \omega \times q(t) = \hat{\omega} q(t) \\ q(0)\text{: Initial coordinates} \end{cases}$$

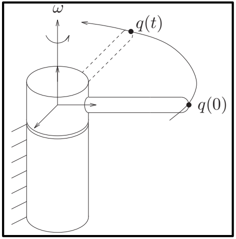

Figure 2.5

2.2 Rotational Motion in $\mathbb{R}^3$

$$R \in SO(3), R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

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$$\begin{cases} \dot{q}(t) = \omega \times q(t) = \hat{\omega} q(t) \\ q(0)\text{: Initial coordinates} \end{cases}$$

$$\Rightarrow q(t) = e^{\hat{\omega} t} q_0 \text{ where } e^{\hat{\omega} t} = I + \hat{\omega} t + \frac{(\hat{\omega} t)^2}{2!} + \frac{(\hat{\omega} t)^3}{3!} + \cdots$$

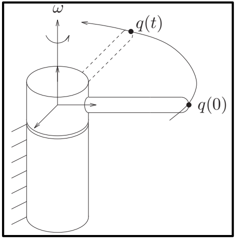

Figure 2.5

2.2 Rotational Motion in $\mathbb{R}^3$

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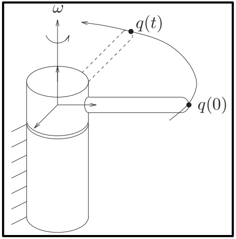

Figure 2.5

$$\begin{cases} \dot{q}(t) = \omega \times q(t) = \hat{\omega} q(t) \\ q(0)\text{: Initial coordinates} \end{cases}$$

$$\Rightarrow q(t) = e^{\hat{\omega} t} q_0 \text{ where } e^{\hat{\omega} t} = I + \hat{\omega} t + \frac{(\hat{\omega} t)^2}{2!} + \frac{(\hat{\omega} t)^3}{3!} + \cdots$$

By the definition of rigid transformation, $R(\omega, \theta) = e^{\hat{\omega}\theta}$. Let $so(3) = \{\hat{\omega} | \omega \in \mathbb{R}^3\}$ or $so(n) = \{S \in \mathbb{R}^{n \times n} | S^T = -S\}$ where $\wedge : \mathbb{R}^3 \mapsto so(3) : \omega \mapsto \hat{\omega}$, we have:

Property 3: $\exp : so(3) \mapsto SO(3), \hat{\omega}\theta \mapsto e^{\hat{\omega}\theta}$

Today

Chapter 2 Rigid Body Motion

2 Rotational motion in $\mathbb{R}^3$

- The *Exponential Map*
- Rodrigues *Formula*
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2.2 Rotational Motion in $\mathbb{R}^3$

Rodrigues' formula ($\|\omega\| = 1$):

$$e^{\hat{\omega}\theta} = I + \hat{\omega}\sin\theta + \hat{\omega}^2(1 - \cos\theta)$$

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Proof :

Let $a \in \mathbb{R}^3$, write

$$a = \omega\theta, \omega = \frac{a}{\|a\|}(\text{or } \|\omega\| = 1), \text{and } \theta = \|a\|$$

$$e^{\hat{\omega}\theta} = I + \hat{\omega}\theta + \frac{(\hat{\omega}\theta)^2}{2!} + \frac{(\hat{\omega}\theta)^3}{3!} + \cdots$$

As

$$\hat{a}^2 = aa^T - \|a\|^2 I, \hat{a}^3 = -\|a\|^2\hat{a}$$

we have:

$$e^{\hat{\omega}\theta} = I + (\theta - \frac{\theta^3}{3!} + \frac{\theta^3}{5!} - \cdots)\hat{\omega} + (\frac{\theta^2}{2!} - \frac{\theta^4}{4!} + \cdots)\hat{\omega}^2$$

$$= I + \hat{\omega}\sin\theta + \hat{\omega}^2(1 - \cos\theta)$$

□

2.2 Rotational Motion in $\mathbb{R}^3$

Rodrigues' formula for $\|\omega\| \neq 1$:

$$e^{\hat{\omega}\theta} = I + \frac{\hat{\omega}}{\|\omega\|}\sin\|\omega\|\theta + \frac{\hat{\omega}^2}{\|\omega\|^2}(1-\cos\|\omega\|\theta)$$

2.2 Rotational Motion in $\mathbb{R}^3$

Rodrigues' formula for $\|\omega\| \neq 1$:

$$e^{\hat{\omega}\theta} = I + \frac{\hat{\omega}}{\|\omega\|} \sin \|\omega\| \theta + \frac{\hat{\omega}^2}{\|\omega\|^2}(1 - \cos \|\omega\| \theta)$$

Proof for Property 3:

Let $R \triangleq e^{\hat{\omega}\theta}$, then:

$$(e^{\hat{\omega}\theta})^{-1} = e^{-\hat{\omega}\theta} = e^{\hat{\omega}^T\theta} = (e^{\hat{\omega}\theta})^T$$

$$\Rightarrow R^{-1} = R^T \Rightarrow R^T R = I \Rightarrow \det R = \pm 1$$

From $\det \exp(0) = 1$, and the continuity of det function w.r.t. θ, we have $\det e^{\hat{\omega}\theta} = 1, \forall \theta \in \mathbb{R}$ $\square$

2.2 Rotational Motion in $\mathbb{R}^3$

Property 4: The exponential map is onto.

2.2 Rotational Motion in $\mathbb{R}^3$

Property 4: The exponential map is onto.

Proof :

Given $R \in SO(3)$, to show $\exists \omega \in \mathbb{R}^3, \|\omega\| = 1$ and θ s.t. $R = e^{\hat{\omega}\theta}$

Let

$$R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

and

$$v_\theta = 1 - \cos\theta, c_\theta = \cos\theta, s_\theta = \sin\theta$$

By Rodrigues' formula $e^{\hat{\omega}\theta} = I + \hat{\omega}\sin\theta + \hat{\omega}^2(1 - \cos\theta)$

$$e^{\hat{\omega}\theta} = \begin{bmatrix} \omega_1^2 v_\theta + c_\theta & \omega_1\omega_2 v_\theta - \omega_3 s_\theta & \omega_1\omega_3 v_\theta + \omega_2 s_\theta \\ \omega_1\omega_2 v_\theta + \omega_3 s_\theta & \omega_2^2 v_\theta + c_\theta & \omega_2\omega_3 v_\theta - \omega_1 s_\theta \\ \omega_1\omega_3 v_\theta - \omega_2 s_\theta & \omega_2\omega_3 v_\theta + \omega_1 s_\theta & \omega_3^2 v_\theta + c_\theta \end{bmatrix}$$

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2.2 Rotational Motion in $\mathbb{R}^3$

Taking the trace of both sides,

$$\mathrm{tr}(R) = r_{11} + r_{22} + r_{33} = 1 + 2\cos\theta = \sum_{i=1}^{3} \lambda_i$$

where λ_i is the eigenvalue of $R, i = 1, 2, 3$

Case 1: $\mathrm{tr}(R) = 3$ or $R = I, \; \theta = 0 \Rightarrow \omega\theta = 0$

2.2 Rotational Motion in $\mathbb{R}^3$

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Case 1: $\mathrm{tr}(R) = 3$ or $R = I,\ \theta = 0 \Rightarrow \omega\theta = 0$

Case 2: $-1 < \mathrm{tr}(R) < 3$,

$$\theta = \arccos\frac{\mathrm{tr}(R) - 1}{2} \Rightarrow \omega = \frac{1}{2s_\theta}\begin{bmatrix} r_{32} - r_{23} \\ r_{13} - r_{31} \\ r_{21} - r_{12} \end{bmatrix}$$

2.2 Rotational Motion in $\mathbb{R}^3$

Taking the trace of both sides,

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$$\begin{aligned}
\text{Case 1:}&\quad \mathrm{tr}(R) = 3 \text{ or } R = I,\ \theta = 0 \Rightarrow \omega\theta = 0 \\
\text{Case 2:}&\quad -1 < \mathrm{tr}(R) < 3, \\
&\quad \theta = \arccos\frac{\mathrm{tr}(R) - 1}{2} \Rightarrow \omega = \frac{1}{2s_\theta}\begin{bmatrix} r_{32} - r_{23} \\ r_{13} - r_{31} \\ r_{21} - r_{12} \end{bmatrix} \\
\text{Case 3:}&\quad \mathrm{tr}(R) = -1 \Rightarrow \cos\theta = -1 \Rightarrow \theta = \pm\pi
\end{aligned}$$

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2.2 Rotational Motion in $\mathbb{R}^3$

Following are 3 possibilities:

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \Rightarrow \omega = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix},$$

$$R = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \Rightarrow \omega = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix},$$

$$R = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \omega = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Note that if $\omega\theta$ is a solution, then $\omega(\theta \pm n\pi), n = 0, \pm 1, \pm 2, \ldots$ is also a solution. □

2.2 Rotational Motion in $\mathbb{R}^3$

Definition: Exponential coordinate

$\omega\theta \in \mathbb{R}^3$, with $e^{\hat{\omega}\theta} = R$ is called the exponential coordinates of R

Exp :

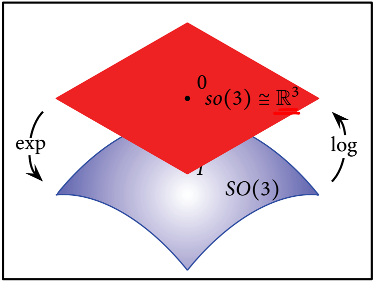

Figure 2.6

2.2 Rotational Motion in $\mathbb{R}^3$

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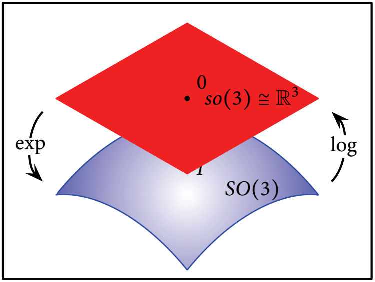

Figure 2.6

Property 5: exp is 1-1 when restricted to an open ball in $\mathbb{R}^3$ of radius π.

2.2 Rotational Motion in $\mathbb{R}^3$

Theorem 1 (Euler):
Any orientation is equivalent to a rotation about a fixed axis $\omega \in \mathbb{R}^3$ through an angle $\theta \in [-\pi, \pi]$.

1707–1783

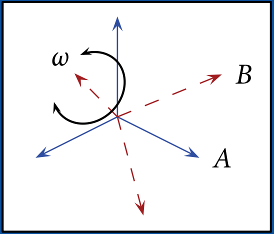

Figure 2.7

$SO(3)$ can be visualized as a solid ball of radius π.

Today

Chapter 2 Rigid Body Motion

2 Rotational motion in $\mathbb{R}^3$

- The *Exponential Map*
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Other Parametrizations of $SO(3)$:

- XYZ fixed angles (or Roll-Pitch-Yaw angle)

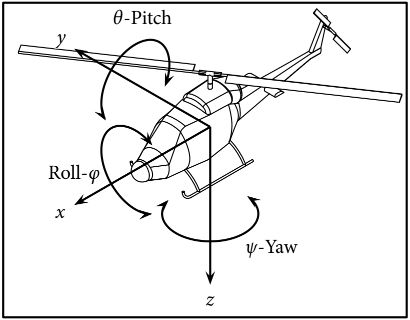

Figure 2.8

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2.2 Rotational Motion in $\mathbb{R}^3$

- *XYZ* fixed angles (or Roll-Pitch-Yaw angle) Continued

$$R_x(\varphi) := e^{\hat{x}\varphi} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi & -\sin\varphi \\ 0 & \sin\varphi & \cos\varphi \end{bmatrix}$$

$$R_y(\theta) := e^{\hat{y}\theta} = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$R_z(\psi) := e^{\hat{z}\psi} = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_{ab} = R_x(\varphi)R_y(\theta)R_z(\psi)$$
$$= \begin{bmatrix} c_\theta c_\psi & -c_\theta s_\psi & s_\theta \\ s_\varphi s_\theta c_\psi + c_\varphi s_\psi & -s_\varphi s_\theta s_\psi + c_\varphi c_\psi & -s_\varphi c_\theta \\ -c_\varphi s_\theta c_\psi + s_\varphi s_\psi & c_\varphi s_\theta s_\psi + s_\varphi c_\psi & c_\varphi c_\theta \end{bmatrix}$$

2.2 Rotational Motion in $\mathbb{R}^3$

- ZYX Euler angle

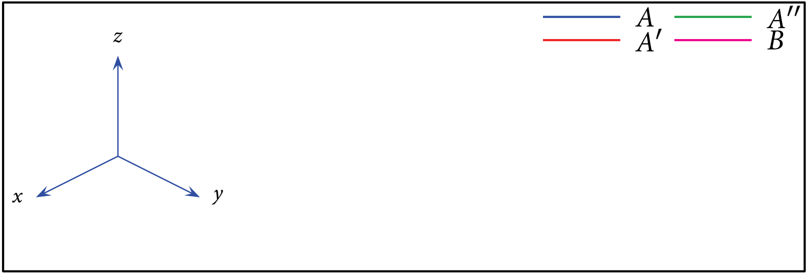

Figure 2.9

2.2 Rotational Motion in $\mathbb{R}^3$

- ZYX Euler angle

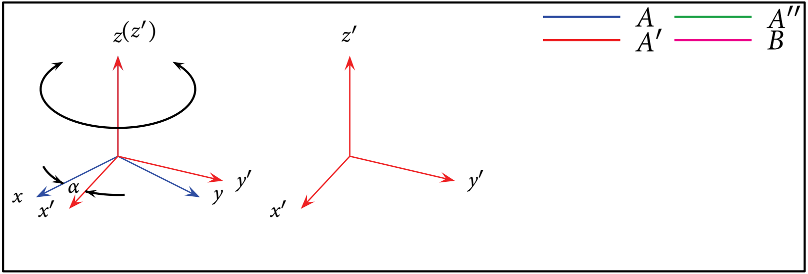

Figure 2.9

$$R_{aa'} = R_z(\alpha)$$

2.2 Rotational Motion in $\mathbb{R}^3$

- ZYX Euler angle

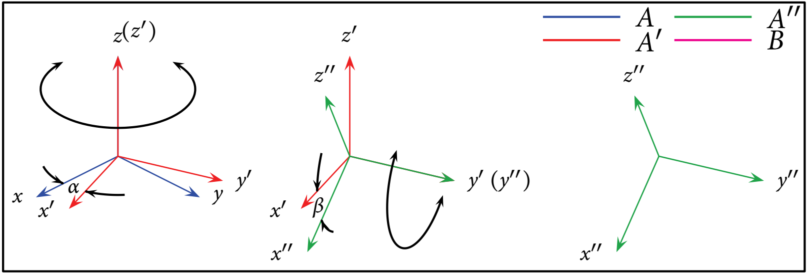

Figure 2.9

$$R_{aa'} = R_z(\alpha) \qquad R_{a'a''} = R_y(\beta)$$

2.2 Rotational Motion in $\mathbb{R}^3$

- ZYX Euler angle

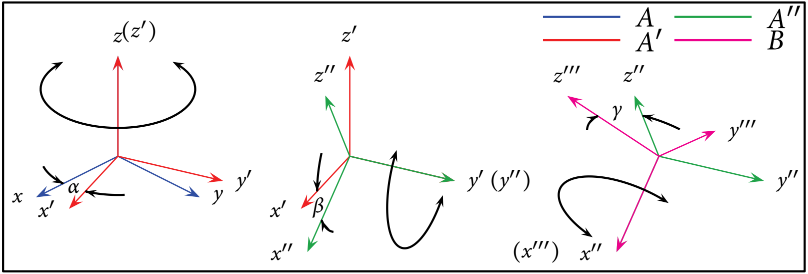

Figure 2.9

$$R_{aa'} = R_z(\alpha) \qquad R_{a'a''} = R_y(\beta) \qquad R_{a''b} = R_x(\gamma)$$

$$R_{ab} = R_z(\alpha)R_y(\beta)R_x(\gamma)$$

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2.2 Rotational Motion in $\mathbb{R}^3$

- *ZYX* Euler angle (continued)

$$R_{ab}(\alpha,\beta,\gamma) = \begin{bmatrix} c_\alpha c_\beta & -s_\alpha c_\gamma + c_\alpha s_\beta s_\gamma & s_\alpha s_\gamma + c_\alpha s_\beta c_\gamma \\ s_\alpha c_\beta & c_\alpha c_\gamma + s_\alpha s_\beta s_\gamma & -c_\alpha s_\gamma + s_\alpha s_\beta c_\gamma \\ -s_\beta & c_\beta s_\gamma & c_\beta c_\gamma \end{bmatrix}$$

Note: When $\beta = \frac{\pi}{2}$, $\cos\beta = 0$, $\alpha + \gamma = \text{const} \Rightarrow$ singularity!

$$\beta = \text{atan2}(-r_{31}, \sqrt{r_{32}^2 + r_{33}^2})$$
$$\alpha = \text{atan2}(r_{21}/c_\beta, r_{11}/c_\beta)$$
$$\gamma = \text{atan2}(r_{32}/c_\beta, r_{33}/c_\beta)$$

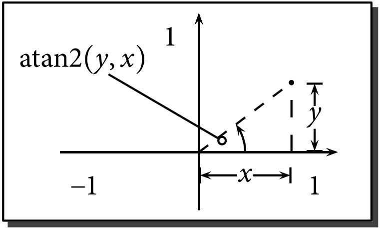
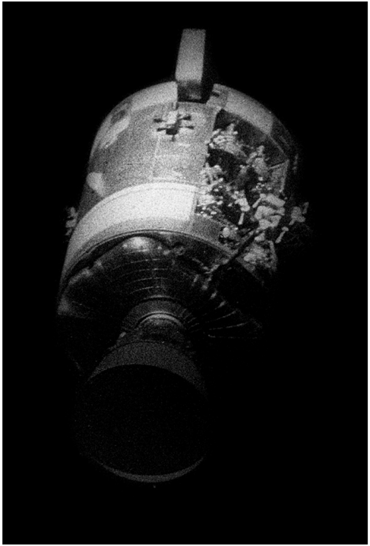


Figure 2.10

Apollo 10




UPLINK ACTY
TEMP
NO ATT
GIMBAL LOCK
STBY
PROG
KEY REL
RESTART
OPR ERR
TRACKER
COMP ACTY
PROG
VERB
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7
8
9
CLR
ENTR
4
5
RSET
NOUN
1
2
3
KEY REL
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ABORT
LV ENGINES
GDC
TRANS CONTR PWR
OFF
LV TANK PRESS
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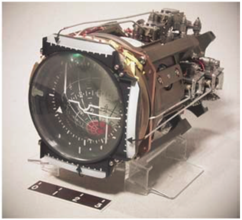
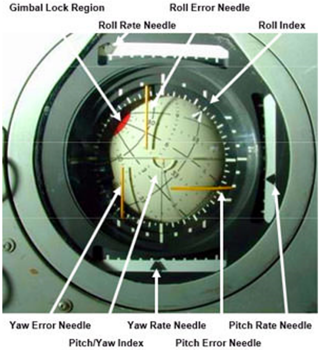
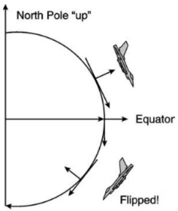



Gimbal Lock Region
Roll Error Needle
Roll Rate Needle
Roll Index
Yaw Error Needle
Yaw Rate Needle
Pitch Rate Needle
Pitch/Yaw Index
Pitch Error Needle

F-16 Fly-By-Wire Fighter Jet



Today

Chapter 2 Rigid Body Motion

2 Rotational motion in $\mathbb{R}^3$

- The *Exponential Map*
- Rodrigues *Formula*
- *Euler Angles*
- *Quaternions*

Ouaternions to the Rescue

Hamilton's Walk

2.2 Rotational Motion in $\mathbb{R}^3$

§ Quaternions:

$Q = q_0 + q_1 i + q_2 j + q_3 k$
where $i^2 = j^2 = k^2 = -1, i \cdot j = k, j \cdot k = i, k \cdot i = j$

§ Quaternions:

$Q = q_0 + q_1 i + q_2 j + q_3 k$
where $i^2 = j^2 = k^2 = -1, i \cdot j = k, j \cdot k = i, k \cdot i = j$

Property 1: Define $Q^* = (q_0, q)^* = (q_0, -q), q_0 \in \mathbb{R}, q \in \mathbb{R}^3$
$\|Q\|^2 = QQ^* = {q_0}^2 + {q_1}^2 + {q_2}^2 + {q_3}^2$

2.2 Rotational Motion in $\mathbb{R}^3$

§ Quaternions:

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Property 2: $Q = (q_0, q), P = (p_0, p)$
$QP = (q_0 p_0 - q \cdot p, q_0 p + p_0 q + q \times p)$

§ Quaternions:

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Property 2: $Q = (q_0, q), P = (p_0, p)$
$QP = (q_0 p_0 - q \cdot p, q_0 p + p_0 q + q \times p)$

Property 3: (a) The set of unit quaternions forms a group
(b) If $R = e^{\hat{\omega}\theta}$, then $Q = (\cos\frac{\theta}{2}, \omega \sin\frac{\theta}{2})$
(c) Q acts on $x \in \mathbb{R}^3$ by QXQ^*, where $X = (0, x)$

□ Unit Quaternions:

Given $Q = (q_0, q), q_0 \in \mathbb{R}, q \in \mathbb{R}^3$, the vector part of QXQ^* is given by $R(Q)x$, recall that

$$q_0 = \cos\frac{\theta}{2}, q = \omega \sin\frac{\theta}{2}$$

and the Rodrigues' formula:

$$e^{\hat{\omega}\theta} = I + \hat{\omega}\sin\theta + \hat{\omega}^2(1-\cos\theta)$$

then

$$\begin{aligned} R(Q) &= I + 2q_0\hat{q} + 2\hat{q}^2 \\ &= \begin{bmatrix} 1-2(q_2^2+q_3^2) & -2q_0q_3+2q_1q_2 & 2q_0q_2+2q_1q_3 \\ 2q_0q_3+2q_1q_2 & 1-2(q_1^2+q_3^2) & -2q_0q_1+2q_2q_3 \\ -2q_0q_2+2q_1q_3 & 2q_0q_1+2q_2q_3 & 1-2(q_1^2+q_2^2) \end{bmatrix} \end{aligned}$$

where $\|Q\| \triangleq q_0^2+q_1^2+q_2^2+q_3^2 = 1$

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□ Quaternions (continued):

Conversion from Roll-Pitch-Yaw angle to unit quaternions:

$$Q = (\cos\frac{\varphi}{2}, x\sin\frac{\varphi}{2})(\cos\frac{\theta}{2}, y\sin\frac{\theta}{2})(\cos\frac{\psi}{2}, z\sin\frac{\psi}{2}) \Rightarrow$$

$$q_0 = \cos\frac{\varphi}{2}\cos\frac{\theta}{2}\cos\frac{\psi}{2} - \sin\frac{\varphi}{2}\sin\frac{\theta}{2}\sin\frac{\psi}{2}$$

$$q = \begin{bmatrix} \cos\frac{\varphi}{2}\sin\frac{\theta}{2}\sin\frac{\psi}{2} + \sin\frac{\varphi}{2}\cos\frac{\theta}{2}\cos\frac{\psi}{2} \\ \cos\frac{\varphi}{2}\sin\frac{\theta}{2}\cos\frac{\psi}{2} - \sin\frac{\varphi}{2}\cos\frac{\theta}{2}\sin\frac{\psi}{2} \\ \cos\frac{\varphi}{2}\cos\frac{\theta}{2}\sin\frac{\psi}{2} + \sin\frac{\varphi}{2}\sin\frac{\theta}{2}\cos\frac{\psi}{2} \end{bmatrix}$$

Conversion from unit quaternions to roll-pitch-yaw angles (?)

† End of Section †

Today

Chapter 2 Rigid Body Motion

2 Rotational motion in $\mathbb{R}^3$

- The *Exponential Map*
- Rodrigues *Formula*
- *Euler Angles*
- *Quaternions*