Practice Midterm II
EECS C 106A/C206A, Fall 2019

Two 8.5 x 11 crib sheets allowed, double sided.

Remember if something is true you have to prove it. If on the other hand something is false, you need to give a counterexample.

Some Notes:

1. This midterm is perhaps a bit longer than the actual midterm (see the other questions thing for ros questions). I also think it’s a little harder than the actual midterm, though of course that’s a matter of opinion.

2. This midterm is not guaranteed to be 100% comprehensive. That said, I think it reasonably spans the actual midterm material.

3. Remember that any material from the class is fair game, be it from lecture, homework, discussion, lab, or the sections of the book that we’ve covered.

Name:

SID:

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Question 1: Jacobian Multiple Choice

Let \( J^*(\theta_1, \theta_2, \theta_3) \) be the manipulator Jacobian of a robot parameterized by the joint position vector \( \theta \). In the current configuration, all joint positions are 0. The Jacobian, joint velocities, and joint torques/forces are:

\[
J^*(0,0,0) = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 1 \\
0 & 1 & 0 \\
0 & 0 & 0 \\
0 & -1 & 0 \\
1 & 0 & 0
\end{bmatrix}
\]

All of the robot’s joints are either revolute or prismatic.

(a) How many joints does this robot have?
- \( \bigcirc 0 \quad \bigcirc 1 \quad \bigcirc 2 \quad \bigcirc 3 \quad \bigcirc 4 \quad \bigcirc 5 \quad \bigcirc 6 \quad \bigcirc 7 \quad \bigcirc 8 \) (1)

(b) How many revolute joints?
- \( \bigcirc 0 \quad \bigcirc 1 \quad \bigcirc 2 \quad \bigcirc 3 \quad \bigcirc 4 \quad \bigcirc 5 \quad \bigcirc 6 \quad \bigcirc 7 \quad \bigcirc 8 \) (1)

(c) This robot is in a singular configuration.
- \( \bigcirc \) True  \( \bigcirc \) False  \( \bigcirc \) Not enough info (2)

(d) In this configuration, in which directions is it possible to induce a nonzero velocity?
- \( \square v_x \quad \square v_y \quad \square v_z \quad \square \omega_x \quad \square \omega_y \quad \square \omega_z \) (2)

(e) In this configuration, in which directions is it possible to induce a nonzero force/torque?
- \( \square f_x \quad \square f_y \quad \square f_z \quad \square \tau_x \quad \square \tau_y \quad \square \tau_z \) (2)

(f) Which column(s) of the spatial Jacobian are constant for all values of \( \theta_1 \)?
- \( \square \) First column  \( \square \) Second column  \( \square \) Third column (1)

(g) Which column(s) of the spatial Jacobian are constant for all values of \( \theta_2 \)?
- \( \square \) First column  \( \square \) Second column  \( \square \) Third column (1)

(h) Which column(s) of the spatial Jacobian are constant for all values of \( \theta_3 \)?
- \( \square \) First column  \( \square \) Second column  \( \square \) Third column (1)

(i) How many singular configurations does this robot have?
- \( \bigcirc 0 \quad \bigcirc 1 \quad \bigcirc 3 \quad \bigcirc 6 \quad \bigcirc 18 \quad \bigcirc 36 \quad \bigcirc \infty \quad \bigcirc \) Not enough info (2)
Question 2: Dynamics ................................................................. 8 points
Consider the mass-pulley system shown in Figure 2 in the Appendix. This system is classically referred to as Atwood’s Machine. The pulley is mounted on a frictionless bearing with a moment of inertia $I$ and a radius $r$. Two masses, $m_1$ and $m_2$, are suspended across the pulley by a rope of fixed length $l$. The position, $x$, is defined as the vertical distance of the mass $m_1$ from the center of the pulley. You may assume that the rope does not slip with respect to the pulley.

(a) Determine the position of the second mass $m_2$ as a function of the position of the first mass $m_1$ with the given parameters. The following equation for the circumference of a circle might be useful:

$$C = 2\pi r$$

(b) Derive an equation for the acceleration of the mass $m_1$ using Lagrangian mechanics. (6)
Question 3: Path Planning

You have a simple 2-dof manipulator shown below in Figure 1

![2-dof manipulator](image)

Figure 1: 2-dof manipulator

This manipulator has one revolute joint $\xi_1$ and one prismatic joint $\xi_2$. It starts in the horizontal configuration $g^i$ ($\theta = [0, l_1]$) and ends in the vertical configuration $g^f$ ($\theta = [\frac{\pi}{2}, l_1]$).

(a) We can define a "straight line path" or constant velocity path of a coordinate $q$ as a path that goes from $q_i$ to $q_f$ at constant velocity. What’s a straight line path in joint space? W and a straight line path in workspace? Draw them.
(b) Assume that you want the robot to complete this trajectory in $T$ seconds. What will your trajectories $\theta(t)$ be for both the joint space and work space trajectories? Note: You shouldn’t need to do any nontrivial integrals.

(c) What is the jacobian of this manipulator at the initial configuration. Assume that the $z$ axis extends out of the page.

(d) How many singularities will this manipulator have? Where will they occur?
Question 4: Physics Review ................................. 10 points
An object of mass $m$ is launched at a velocity of $v$ m/s at some angle $\theta$ from horizontal. The initial position of the object is $x = 0$, $y = 0$. The ground is at $y = 0$ for all $x$. (See figure 3)
(a) What are the equations of motion for the mass? (use cartesian coordinates) (2)
(b) What is the maximum height reached by the object? (2)
(c) What is the total flight time? (2)
(d) How far does the object travel? (2)
(e) At what velocity does the mass strike the ground? (2)
Question 5: Singular Configuration .............................................. 12 points
Three revolute joint axes with twists $\xi_i = [q_i \times \omega_i, \omega_i]^T, i = 1, 2, 3$ are said to be parallel if
$$\omega_i = \pm \omega_j, \ i, j = 1, 2, 3$$
A prismatic joint with twist $\xi_4 = [v, 0]^T$ is said to be perpendicular to these revolute joints if
$$v^T \omega_i = 0, \ i = 1, 2, 3$$
Show that a six degree of freedom manipulator with three parallel revolute axes and a prismatic axis perpendicular to all three is at a singular configuration, that is, that $J(\theta)$ is singular.
1 Appendix: Figures and Useful Formulae

Figure 2: Atwood’s Machine

Figure 3: Projectile motion