Hand Dynamics and Control

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Outline

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Lagrangian Equations with Constraints

Hand Dynamics

Hand Control

Pfaffian Constraints

A constraint on a mechanical system limits its motion. If the configuration denoted by $q \in \Re^n$, the simplest form of constraints are the so-called **holonomic constraints**

$$h_i(q) = 0 \qquad i = 1, \ldots, k$$

These constrain q to lie in a manifold of dimension $n - k$. Other systems are characterized by having constraints on their velocities. A set of constraints of the form

$$A(q)\dot{q} = 0$$

with $A(q) \in \Re^{p \times n}$ is referred to as a system of **Pfaffian constraints.** We will assume that the constraints are independent by asking that the rows of $A(q)$ be linearly independent at q.

Holonony, Nonholonomy

Since, the first set of equations may be differentiated to get

$$dh(q)\dot{q} = 0$$

which is clearly of the form

$$A(q)\dot{q} = 0$$

we may ask if all Pfaffian contraints are equivalent to non-holonomic systems. The answer to this question is quite subtle and is the subject of Chapters 7 and 8 of MLS on holonomy, non-holonomy, integrability, etc.
However, amazingly the discussion of the dynamics of constrainted Pfaffian systems does not need us to know if the Pfaffian system is holonomic or not!

D'Alembert Forces, Lagrange Multipliers

We add an assumption that the constraint forces on the body dynamics which enforce the Pfaffian constraints do not do any work. This is known as **d'Alembert's principle** and says that the forces of constraint $\Gamma \in \Re^n$ associated with a Pfaffian constraint of the form

$$A(q)\dot{q} = 0$$

with $A(q) \in \Re^{p \times n}$ are of the form

$$\Gamma = A^T(q)\lambda$$

for a suitably chosen *Lagrange multiplier* $\lambda \in \Re^p$. Indeed it is easy to see that

$$\Gamma^T \dot{q} = \lambda^T A(q)\dot{q} = 0$$

establsihing that the power exerted by Γ is zero.

Lagrange D'Alembert Equations of Motion

The Lagrange D'Alembert equations of motion are

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = (\Upsilon - A^T(q)\lambda)^T$$

where $\Upsilon \in \Re^n$ represents the externally applied forces. The issue with this equation is that the Lagrange multiplier λ is unknown.

Lagrange D'Alembert for Robots

We will work through the details by using for definiteness the Lagrangian for a robot (or a set of linked rigid bodies)

$$L(q,\dot{q}) = \frac{1}{2}\dot{q}^T M(q)\dot{q} - V(q)$$

where $M(q) \in \Re^{n \times n}$ is the moment of inertia and $V(q)$ is the potential energy yielding the Lagrange D'Alembert equation

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + N(q) + A^T(q)\lambda = F$$

where F is the external force and

$$C_{ij}(q,\dot{q}) = \frac{1}{2}\sum_{k=1}^{n}\left(\frac{\partial M_{ij}}{\partial \theta_k} + \frac{\partial M_{ik}}{\partial \theta_j} - \frac{\partial M_{kj}}{\partial \theta_i}\right)\dot{\theta}_k \qquad N_i(q) = -\frac{\partial V}{\partial q_i}$$

Differentiating the Pfaffian constraint equation $A(q)\dot{q} = 0$ yields

$$A(q)\ddot{q} + \dot{A}(q)\dot{q} = 0$$

Solving for the Lagrange Multiplier

Solve for $\ddot{q}$ from the Lagrange d'Alembert equation and use it in the differentiated constraint to get

$$A(q)M^{-1}(q)\left(F - C(q,\dot{q})\dot{q} - N(q) - A^T(q)\lambda\right) + \dot{A}(q)\dot{q} = 0$$

If the "modified" moment of inertia matrix

$$A(q)M^{-1}(q)A^T(q) \in \Re^{p\times p}$$

is invertible then the Lagrange multiplier can be solved to be

$$\lambda = (AM^{-1}A^T)^{-1}(AM^{-1}(F - C\dot{q} - N) + \dot{A}\dot{q})$$

This in turn is used in the Lagrange D'Alembert equation to give the constrained system dynamics.

Lagrange D'Alembert Alternatively

Partition the constraint matrix $A(q)$ into $[A_1(q)A_2(q)]$ with $A_2(q) \in \Re^{p\times p}$ of full rank. Also partition $q^T = (q_1^T q_2^T)$ compatibly. With respect to this partition, we get

$$\delta q_2 = A_2^{-1}(q)A_1(q)\delta q_1$$

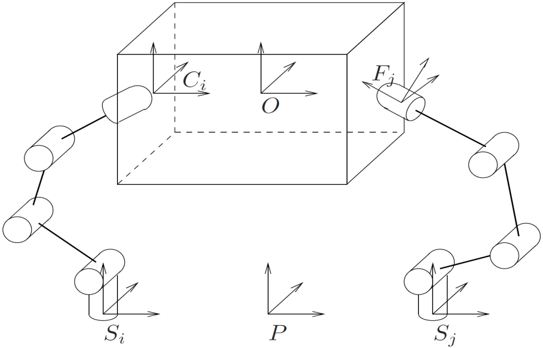
and the Lagrange d'Alembert equations can be rewritten as

$$\left(\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_1} - \frac{\partial L}{\partial q_1} - \Upsilon_1^T\right) - A_1^T A_2^{-T}\left(\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_2} - \frac{\partial L}{\partial q_2} - \Upsilon_2^T\right) = 0^T$$

Using $\dot{q}_2 = A_2^{-1}A_1\dot{q}_1$ we can eliminate $\dot{q}_2, \ddot{q}_2$ from the preceding equation

Outline

Robot Hand Dynamics



A multi-fingered robot hand has k fingers with joint angles $\theta_{f_1}, \ldots \theta_{f_k} \in \Re^n$ Each of the fingers is a robot in its own right so the composite dynamics are

$$M_f(\theta)\ddot{\theta} + C_f(\theta, \dot{\theta})\dot{\theta} + N_f(\theta) = \tau$$

Body Dynamics

The object being grasped can also be modeled as above with $x \in \Re^6$ being a local parameterization of the SE(3) of the body to get

$$M_o(x)\ddot{x} + C_o(x,\dot{x}) + N_o(x) = 0$$

The grasp equations connect the finger $\dot{\theta}$ and the the body $\dot{x}$ as

$$J_h(\theta,x)\dot{\theta} = G^T(\theta,x)\dot{x}$$

To combine these into the Lagrange d'Alembert equations of the whole system, we assme

1. The grasp is force closure and manipulable
2. The hand Jacobian is invertible (the hand as many degrees of freedom as are needed to maniuplate the object).
3. The contact forces lie in the FC and cotact is maintained.

Assumptions 1 and 2 may be relaxed, but not Assumption 3.

Hand and Body Dynamics

The composite Lagrangian is

$$L = \frac{1}{2}\dot{\theta}^T M_f \dot{\theta} + \frac{1}{2}\dot{x}^T M_o \dot{x} - V_f(\theta) - V_o(x)$$

Because of assumption 1, we can solve for $\dot{\theta}$ in the grasp constraint as

$$\dot{\theta} = J_h^{-1} G^T \dot{x}$$

to get the Lagrange d'Alembert equations of motion as

$$\left(\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x}\right) + G J_h^{-T}\left(\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} - \frac{\partial L}{\partial \theta}\right) = G J_h^{-T} \boldsymbol{\tau}$$

We can simplify this using the grasp constraint to eliminate $\dot{\theta}, \ddot{\theta}$.

Outline

Hand Control Equations

The overall Lagrange d'Alembert equations are

$$\tilde{M}(q)\ddot{x} + \tilde{C}(q,\dot{q})\dot{x} + \tilde{N}(q) = F$$

where

$$\begin{aligned}
\tilde{M} = &\quad M_o + GJ_h^{-T} M_f J_h^{-1} G^T \\
\tilde{C} = &\quad C_o + GJ_h^{-T}\left(C_f J_h^{-1} G^T + M_f \tfrac{d}{dt}\left(J_h^{-1} G^T\right)\right) \\
\tilde{N} = &\quad N_o + GJ_h^{-T} N_f \\
F = &\quad GJ_h^{-T}\tau
\end{aligned}$$

Now it should come as no surprise that

1. $\tilde{M}(q)$ is symmetric and positive definite
2. $\dot{\tilde{M}}(q) - 2\tilde{C}(q,\dot{q})$ is skew symmetric

Hand Control

The composite equations of motion are

$$\tilde{M}(q)\ddot{x} + \tilde{C}(q,\dot{q})\dot{x} + \tilde{N} = F = GJ_h^{-T}\tau$$

The control task is to get $x(\cdot)$ to track $x_d(\cdot)$, the trajectory of the grasped object. Here is the computed torque controller for the hand dynamics

$$F_c = \tilde{M}\left(\ddot{x}_d - K_v\dot{e} - K_p e\right) + \tilde{C}(q,\dot{q})\dot{x} + \tilde{N}(q)$$

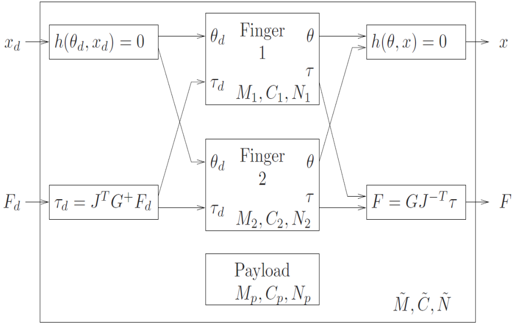
You might ask, how does one compute the finger torques $\tau \in \Re^p$. Because the grasp is force closure G is surjective. Any finger torques $J_h^{-T}\tau$ that lie in the null space of G though will not produce any motions, since they provide only internal forces. Internal forces may well be needed to make sure that the contact forcs lie in the friction cone. Thus, we choose

$$\tau = J_h^T\left(G^\dagger F_c + F_n\right)$$

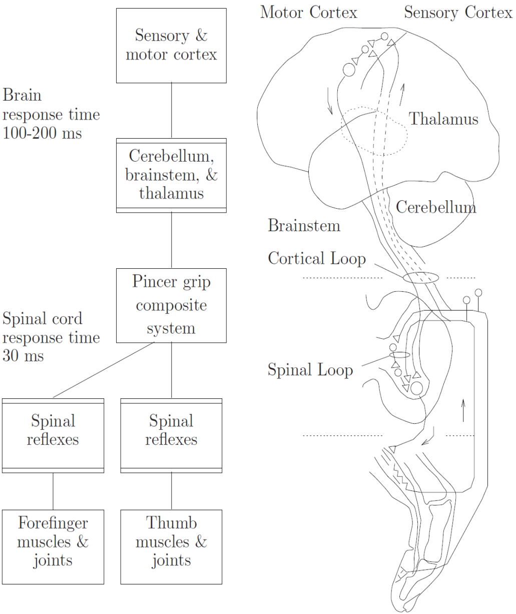
where $G^\dagger = G^T(GG^T)^{-1}$ and F_n lies in the null space of G and is chosen to make sure that the grasp is maintained.

Other hand controllers

Just like robots, we can have PD hand controllers as well as computer torque hand controllers and so on. The following figure gives you an idea of the data flow for computing the controllers



Biological Motor Control



Conjectured Biological Hand Controller