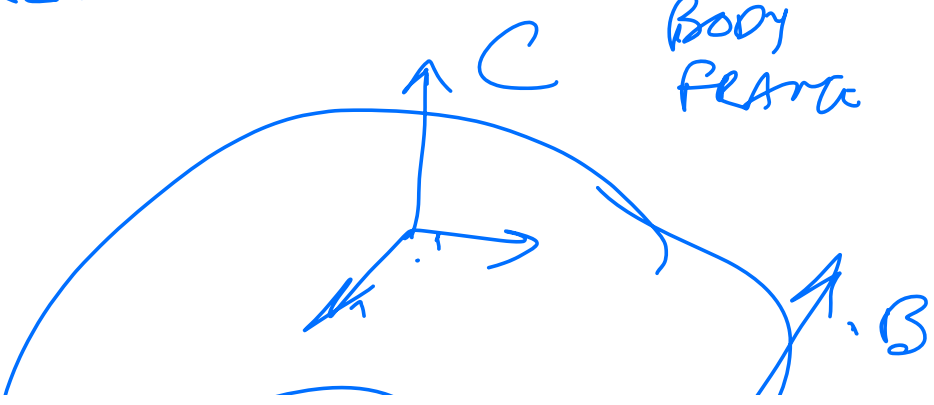
TI DAY 3/14/23

EECS 106B/206B

REVIEW OF TWISTS



$$g_{ab} = \begin{bmatrix} R_{ab} & p_{ab} \\ 0\,0\,0 & 1 \end{bmatrix} \in SE(3) \quad \mathbb{R}^{4\times 4}$$

$$g_{ab}\, g_{bc} = g_{ac}$$

"GOW" VELOCITY

$$\hat{V}^b_{ab} = g_{ab}^{-1}(t)\, \dot{g}_{ab}(t) \in \mathbb{R}^{4\times 4}$$

$$\begin{bmatrix} \hat{\omega}^b_{ab} & v^b_{ab} \\ 0\,0\,0 & 0 \end{bmatrix} = g_{ab}^{-1}\, \dot{g}_{ab}$$

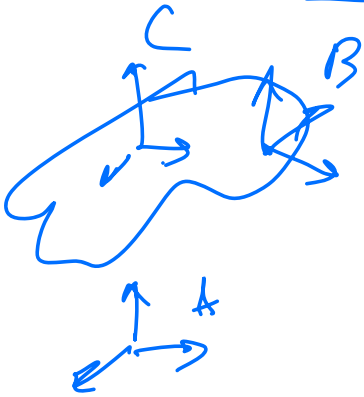
TWIST

$\hat{\omega}^b_{ab} \in SO(3)$ $\qquad \hat{\omega}_{ab} x = \omega_{ab} \times x$

$$V^b_{ab} = \begin{bmatrix} v^b_{ab} \\ \omega^b_{ab} \end{bmatrix} \in \mathbb{R}^6$$

TWIST COORDS.

RELATE V^b_{ab} $\longleftrightarrow$ V^b_{ac}



g_{bc} is NOT A FUNCTION OF TIME

$$g_{ab} = g_{ac} g_{cb}$$

$$\dot{g}_{ab} = \dot{g}_{ac} g_{cb} + g_{ac} \dot{g}_{cb}$$

$$\dot{g}_{ab} = \dot{g}_{ac} g_{cb}$$

$$g^{-1}_{ab} \dot{g}_{ab} = g^{-1}_{ab} \dot{g}_{ac} g_{cb}$$

$$\hat{V}^{b}_{ab} = g_{ab}^{-1} \dot{g}_{ac} \, g_{ac}^{-1} \dot{g}_{ac} \, g_{cb}$$

$$= g_{ab}^{-1} g_{ac} \, \hat{V}^{b}_{ac} \, g_{cb}$$

$$= (g_{ac}^{-1} g_{ab})^{-1} \hat{V}^{b}_{ac} \, g_{cb}$$

$$= (g_{ca} g_{ab})^{-1} \hat{V}^{b}_{ac} \, g_{cb}$$

$$\Rightarrow \hat{V}^{b}_{ab} = g_{cb}^{-1} \hat{V}^{b}_{ac} \, g_{cb} \Leftarrow$$

$$= \begin{bmatrix} R_{cb}^{T} & -R_{cb}^{T} p_{cb} \\ 0 & 1 \end{bmatrix} \hat{V}^{b}_{ac} \begin{bmatrix} R_{cb} & p_{cb} \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \hat{\omega}^{b}_{ab} & v^{b}_{ab} \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} R_{cb}^{T} & -R_{cb}^{T} p_{cb} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{\omega}^{b}_{ac} & v^{b}_{ac} \\ 0 & 0 \end{bmatrix}$$

MULTIPLY BY $\begin{bmatrix} R_{cb} & p_{cb} \\ 0 & 1 \end{bmatrix}$

$$\begin{bmatrix} \hat{\omega}_{ab}^{b} = R_{cb}^{T} \hat{\omega}_{ac}^{b} R_{cb} \\ v_{ab}^{b} = R_{cb}^{T} v_{ac}^{b} + \hat{p}_{bc} R_{cb}^{T} v_{ac}^{b} \end{bmatrix}$$

$$\omega_{ab}^{l} = R_{cb}^{T} \omega_{ac}^{b} = R_{bc} \omega_{ac}^{b}$$

$$\underline{v}_{ab}^{l} = R_{cb}^{T} v_{ac}^{b} + \hat{p}_{bc} R_{cb}^{T} \omega_{ac}^{b}$$

$$= R_{bc} v_{ac}^{b} + \hat{p}_{bc} R_{bc} \omega_{ac}^{b}$$

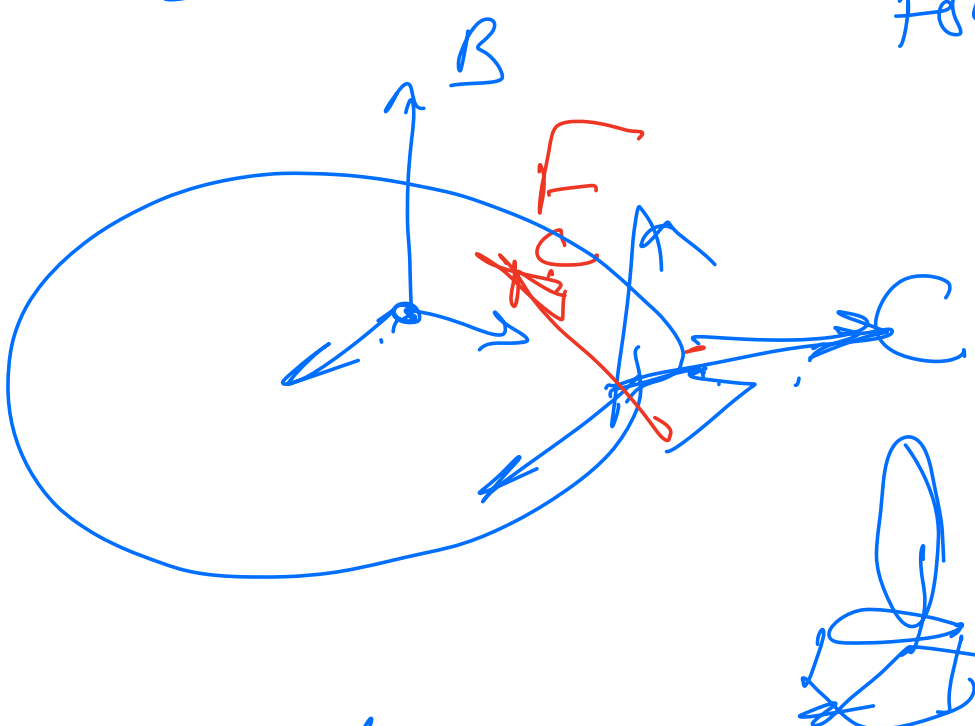
$$\begin{bmatrix} \dot{v}_{ab}^{b} \\ \omega_{ab}^{b} \end{bmatrix} = \underbrace{\begin{bmatrix} R_{bc} & \hat{p}_{bc} R_{bc} \\ 0 & R_{bc} \end{bmatrix}}_{\in \mathbb{R}^{6\times 6}} \begin{bmatrix} v_{ac}^{b} \\ \omega_{ac}^{b} \end{bmatrix}$$

$$V_{ab}^{b} = Ad_{g_{bc}} V_{ac}^{b}$$

TWIST — DUAL — WRENCH

$\begin{bmatrix} v \\ \omega \end{bmatrix}$ GEN VEL

$\begin{bmatrix} f \\ \tau \end{bmatrix}_F$ GEN FORCE



$$\langle F_c, V_{ac}^{b} \rangle = \langle F_b, V_{ab}^{b} \rangle$$

POWER in a frame

POWER in Body

$$F_c^T V_{ac}^b = F_b^T V_{ab}^b$$

$\mathbb{R}^6 \quad \mathbb{R}^6$

$$V_{ab}^b = Ad_{g_{bc}} V_{ac}^b$$

$$F_c^T \left(V_{ac}^b \right) = F_b^T \left(Ad_{g_{bc}} V_{ac}^b \right)$$

$\forall V_{ac}^b \in \mathbb{R}^6$

$$F_c^T = F_b^T Ad_{g_{bc}}$$

$$F_c = Ad_{g_{bc}}^T F_b$$

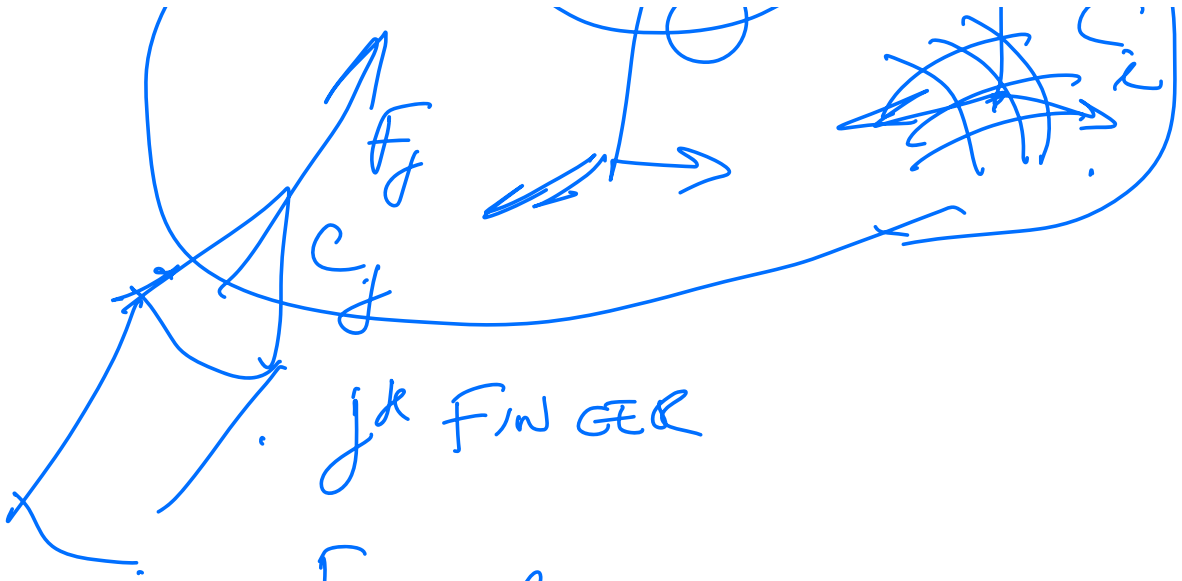
6×6

$$Ad_{g_{bc}} = \begin{bmatrix} R_{bc} & \hat{p}_{bc} R_{bc} \\ 0 & R_{bc} \end{bmatrix}^T$$

$$Ad_{g_{bc}}^T = \begin{bmatrix} R_{bc} & \hat{p}_{bc} R_{bc} \\ 0 & R_{bc} \end{bmatrix}^T$$

$$= \begin{bmatrix} R_{bc}^T & 0 \\ -R_{bc}^T \hat{p}_{bc} & R_{bc}^T \end{bmatrix}$$

$$F_c = Ad_{g_{bc}}^T F_b$$



j^{th} FINGER

RIGID FINGER

P_j CONTACTS W/O FRICTION

$$F_j = B_j f_{cj} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ \hline 0 \\ 0 \\ 0 \end{bmatrix} f_{cj} \left\{ \begin{matrix} \\ \\ \\ \\ \\ \text{TORQUES} \end{matrix} \right.$$

$f_{cj} \geq 0$ $\quad 6 \times 1$

P_j CONTACTS W/ FRICTION

$$F_j = B_j f_{cj} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} f_{cj}$$

$f_{cj} \in \mathbb{R}^3$ $\quad 6 \times 3$

$f_{cj3} \geq 0 \qquad \sqrt{f_{cj1}^2 + f_{cj2}^2} \leq \mu \cdot f_{cj3}$

COEFF OF FRICTION

SOFT FINGER CONTACTS

$f_{cj} \in \mathbb{R}^4$

6×4

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} f_{cj}$$

B_j

$$\sqrt{f_{cj1}^2 + f_{cj2}^2} \leq \mu f_{cj3}.$$

$$f_{cj3} \geq 0$$

$\delta c_{j4} \leq \gamma \, \delta c_{j3}$.

F_1 pt. w/o finch

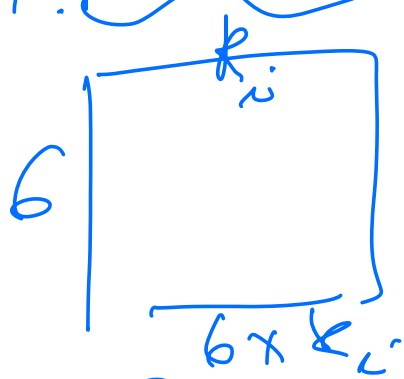
F_2 pt w finch

F_k

$$F_i = B_{c_i} \delta c_i$$

$$F_0 = \overline{Ad^T_{g_{c_i 0}} F_{c_i}} = Ad^T_{g_{c_i 0}} B_i \, \delta c_i$$

$$F_o = \sum_{i=1}^{k} Ad_{g_{ci}}^{T} B_i f_{ci}$$



k_i type of finger

$$G_i \in \mathbb{R}^{6 \times k_i}$$

$$F_o = \underbrace{\begin{bmatrix} G_1 & G_2 & \cdots & G_k \end{bmatrix}}_{G} \begin{bmatrix} f_{c1} \\ f_{c2} \\ \vdots \\ f_{ck} \end{bmatrix}$$

GRASP MATRIX

$f_{ci} \in \mathbb{R}^{k_i}$