EECS 106B/206B

Lecture 6 02/02/2023.

MULTI INPUT MULTI OUTPUT SYSTEMS (MIMO)

TWO INPUT TWO OUTPUT SYSTEMS (TITO)

$x \in \mathbb{R}^n$

$$\dot{x} = f(x) + g_1(x)u_1 + g_2(x)u_2$$

$$f(x) = \begin{bmatrix} f_1(x) \\ \vdots \\ f_n(x) \end{bmatrix} \qquad g_1(x) = \begin{bmatrix} g_{11}(x) \\ g_{12}(x) \\ \vdots \\ g_{1n}(x) \end{bmatrix}$$

\# of entities that can be controlled = # of inputs

$y_1 = h_1(x)$

$y_2 = h_2(x)$

Feedback Linearization of TITO systems

$$\dot y_1 = \frac{d}{dt} h_1(x) = Dh_1(x)\,\dot x$$

$$= Dh_1(x)\left[f(x) + g_1(x)u_1 + g_2(x)u_2\right]$$

$$\dot y_1 = L_f h_1(x) + L_{g_1} h_1(x) u_1 + L_{g_2} h_1(x) u_2$$

If $L_{g_1} h_1(x) = L_{g_2} h_1(x) \equiv 0$

$$\dot y_1 = L_f h_1(x)$$

Def Let r_1 be the smallest integer such that $\dot y_1, \ddot y_1, \dots, y_1^{(r_1-1)}$ do not depend on either u_1 or u_2

ie $\quad L_{g_1} h_1 = L_{g_2} h_1 \equiv 0$

$L_{g_1} L_f h_1 = L_{g_2} L_f h_1 \equiv 0$

$L_{g_1} L_f^{r_1-2} h_1 = L_{g_2} L_f^{r_1-2} h_1 \equiv 0$

$$y_1^{(r_1)} = L_f^{r_1} h_1 + L_{g_1} L_f^{r_1-1} h_1 u_1 + L_{g_2} L_f^{r_1-1} h_2 u_2$$

$\neq 0$

$y_1 = h_1$

$\dot{y}_1 = L_f h_1 + \cancel{L_{g_1} h_1 u_1} + \cancel{L_{g_2} h_1 u_2}$

$\ddot{y}_1 = L_f(L_f h_1) + \cancel{L_{g_1}(L_f h_1) u_1} + \cancel{L_{g_2}(L_f h_1) u_2}$

$\dot{y}_1 = D h_1(x) \left[f(x) + g_1(x) u_1 \right.$

$\left. \cancel{+ g_2(x) u_2} \right]$

y_1

$$= L_f h_1 + L_{g_1} h_1(x) u_1 + L_{g_2} h_1(x) u_2$$

Let r_2 be the corresponding relative degree for y_2

$$y_2 = h_2(x)$$

$$\dot{y}_2 = L_f h_2(x) + \cancel{L_{g_1} h_2(x) u_1} + \cancel{L_{g_2} h_2(x) u_2}$$

$$\ddot{y}_2$$

$$y_2^{(r_2)} = L_f^{r_2} h_2(x) + L_{g_1} L_f^{r_2-1} h_2(x) u_1 + L_{g_2} L_f^{r_2-1} h_2(x) u_2$$

$$\begin{bmatrix} y_1^{(r_1)} \end{bmatrix} = \begin{bmatrix} L_f^{r_1} h_1(x) \end{bmatrix}$$

$$\begin{bmatrix} y_1^{(r_1)} \\ y_2^{(r_2)} \end{bmatrix} = \begin{bmatrix} L_f^{r_1} h_1(x) \\ L_f^{r_2} h_2(x) \end{bmatrix} +$$

$$\begin{bmatrix} L_{g_1} L_f^{r_1-1} h_1 & L_{g_2} L_f^{r_1-1} h_1 \\ L_{g_1} L_f^{r_2-1} h_2 & L_{g_2} L_f^{r_2-1} h_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

2×2

$$\begin{bmatrix} y_1^{(r_1)} \\ y_2^{(r_2)} \end{bmatrix} = \begin{bmatrix} L_f^{r_1} h_1 \\ L_f^{r_2} h_2 \end{bmatrix} + \begin{bmatrix} A(x) \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$A_{ij}(x) = L_{g_j} L_f^{r_i - 1} h_i$$

If $A^{-1}(x)$ exist $\quad \det A(x) \neq 0$

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = A^{-1}(x) \left(\begin{bmatrix} -L_f^{r_1} h_1 \\ -L_f^{r_2} h_2 \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \right)$$

$$y_1^{(r_1)} = v_1$$ decoupling

$$y_2^{(r_2)} = v_2$$

TITO is said to have the rel. degree (r_1, r_2) if

$$L_{g_1} h_1 = L_{g_1} L_f h_1 = \dots L_{g_1} L_f^{r_1-2} h_1 \equiv 0$$

$$L_{g_2} h_1 = L_{g_2} L_f h_1 = \dots = L_{g_2} L_f^{r_1-2} h_1 \equiv 0$$

& either $L_{g_1} L_f^{r_1-1} h_1$ or $L_{g_2} L_f^{r_1-1} h_1 \neq 0$

$$L_{g_1} h_2 = L_{g_1} L_f h_2 = \cdots = L_{g_1} L_f^{n_2-1} h_2 \equiv 0$$

$$L_{g_2} h_2 = L_{g_2} L_f h_2 = \cdots = L_{g_2} L_f^{n_2-1} h_2 \not\equiv 0$$

$$L_{g_1} L_f^{n_2-1} h_2 \text{ or } L_{g_2} L_f^{n_2-1} h_2 \neq 0$$

&

$$A(x) = \begin{bmatrix} L_{g_1} L_f^{n_1-1} h_1 & L_{g_2} L_f^{n_1-1} h_1 \\ L_{g_1} L_f^{n_2-1} h_2 & L_{g_2} L_f^{n_2-1} h_2 \end{bmatrix}$$

is nonsingular

$$\left\{ \begin{array}{l} y_1^{(n_1)} = v_1 \\ y_2^{(n_2)} = v_2 \end{array} \right.$$

$$r_1 + r_2 \leq n$$

$$n - (r_1 + r_2)$$

$$\begin{cases} \xi^1_1 = y_1 \quad \xi^1_2 = \dot{y}_1 \quad \cdots \quad \xi^1_{r_1-1} = y_1^{(r_1-1)} \\ \xi^2_1 = y_2 \quad \xi^2_2 = \dot{y}_2 \quad \cdot \quad \xi^2_{r_2-1} = y_2^{(r_2-1)} \end{cases}$$

$$\boxed{\eta \in \mathbb{R}^{n-r_1-r_2}}$$