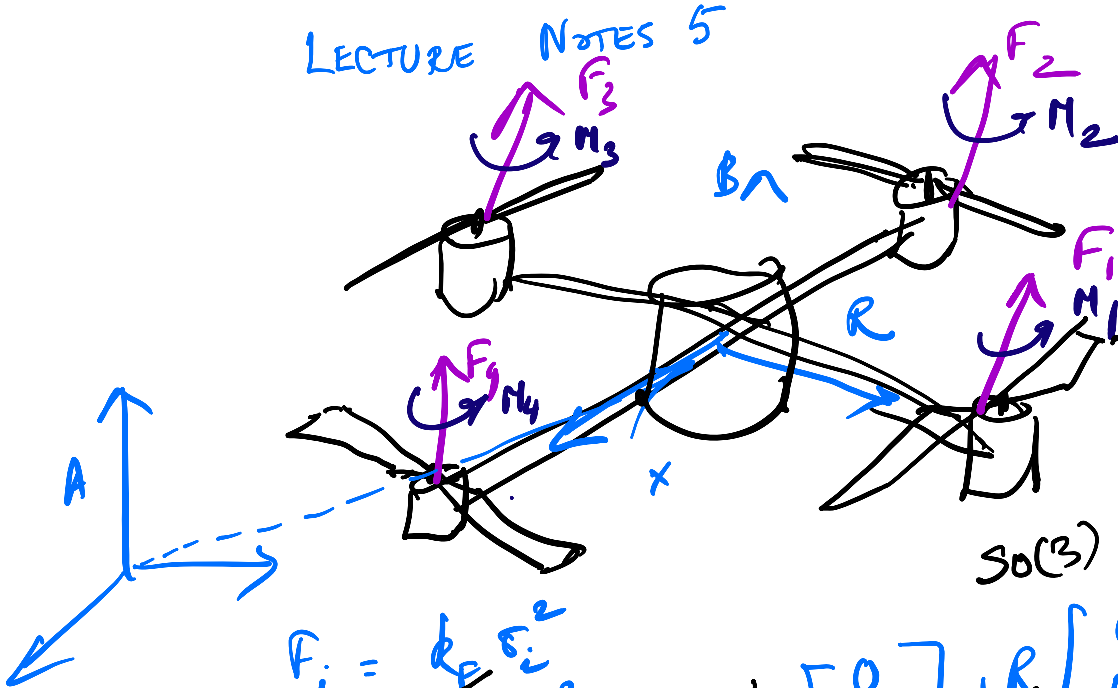
Lecture Notes 5



$$F_i = k_F \omega_i^2$$

$$M_i = k_M \omega_i^2$$

$$m\ddot{x} = \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} + R \begin{bmatrix} 0 \\ 0 \\ \underbrace{F_1 + F_2 + F_3 + F_4} \end{bmatrix}$$

$$\gamma = \frac{k_M}{k_F}$$

$$\dot{R} = R\hat{\omega}$$ Body Ang Vel.

$$I\dot{\omega} + \omega \times I\omega = \begin{bmatrix} L(F_2 - F_4) \\ L(F_3 - F_1) \\ M_1 - M_2 + M_3 - M_4 \end{bmatrix}$$

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & L & 0 & -L \\ -L & 0 & L & 0 \\ \gamma & -\gamma & \gamma & -\gamma \end{bmatrix} \begin{bmatrix} k_p \omega_1^2 \\ k_p \omega_2^2 \\ k_p \omega_3^2 \\ k_p \omega_4^2 \end{bmatrix}$$ ✓ ✓ ✓ ✓

$\sigma_1, \sigma_2 \sigma_3, \sigma_4$ controlled by motor servos

$$m\ddot{x} = \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} + R \begin{bmatrix} 0 \\ 0 \\ u_1 \end{bmatrix}$$

$$\dot{R} = R\hat{\omega}$$

$$I\dot{\omega} = -\omega \times I\omega + \begin{bmatrix} u_2 \\ u_3 \\ u_4 \end{bmatrix}$$

$$R = \underset{\text{YAW}}{e^{\hat{z}\psi}} \; \underset{\text{PITCH}}{e^{\hat{y}\theta}} \; \underset{\text{ROLL}}{e^{\hat{x}\phi}}$$

$$\hat{\omega} = R^T\dot{R} = \hat{z}\dot{\psi} + e^{-\hat{z}\psi}\hat{y}e^{\hat{z}\psi}\dot{\theta} + e^{-\hat{z}\psi}e^{-\hat{y}\theta}\hat{x}e^{\hat{y}\theta}e^{\hat{z}\psi}$$

JACOBIAN

$$\begin{bmatrix} \dot{\psi} \\ \dot{\theta} \\ \dot{\phi} \end{bmatrix} = J(\psi, \theta, \phi)\, \omega$$

OUTPUTS

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ \psi \end{pmatrix}$$

POSITION

YAW

$$\begin{bmatrix}\ddot y_1\\ \ddot y_2\\ \ddot y_3\end{bmatrix} = \begin{bmatrix}0\\ 0\\ -g\end{bmatrix} + R\begin{bmatrix}0\\ 0\\ u_1/m\end{bmatrix}$$

Call $\frac{u_1}{m} = v_1$

$$\dot y_4 = \dot\psi = [1\ 0\ 0]\, J\omega$$

$$\ddot y_4 = [1\ 0\ 0]\,\dot J\omega + [1\ 0\ 0]\, J\dot\omega$$

$$= [1\ 0\ 0]\,\dot J\omega + [1\ 0\ 0]\, J I^{-1}\left(-\omega\times I\omega + \begin{bmatrix}u_2\\ u_3\\ u_4\end{bmatrix}\right)$$

$$\ddot y_4 = xxxx + [1\ 0\ 0]\, J I^{-1}\begin{bmatrix}u_2\\ u_3\\ u_4\end{bmatrix}$$

$$= xxxx + [a_{42}\ a_{43}\ a_{44}]\begin{bmatrix}u_2\\ u_3\\ v_4\end{bmatrix}$$

$$\begin{bmatrix}\ddot y_1\\ \ddot y_2\\ \ddot y_3\end{bmatrix} = xxx + R\begin{bmatrix}0\\ 0\\ 1\end{bmatrix}u_1$$

Call $I^{-1}\begin{bmatrix}u_2\\ u_3\\ u_4\end{bmatrix} = \begin{bmatrix}u_2\\ u_3\\ u_4\end{bmatrix}$

$$\ddot\psi \quad \begin{bmatrix}\ddot y_1\\ \ddot y_2\\ \ddot y_3\\ \ddot y_4\end{bmatrix} = xxx + \begin{array}{c|c} & 4 \\ \hline 4 & \begin{bmatrix} R_z & O \\ 0 & \underset{(\phi,\theta,\psi)}{a_{42}\ a_{43}\ a_{44}} \end{bmatrix}\end{array}\begin{bmatrix}u_1\\ u_2\\ u_3\\ u_4\end{bmatrix}$$

$\boxed{\text{Rank} = 2}$!

Define $u_1 = z_1$ make u_1 a state $R\begin{bmatrix}0\\0\\1\end{bmatrix} = r_3$

$$\begin{bmatrix} y_1^{(3)} \\ y_2^{(3)} \\ y_3^{(3)} \end{bmatrix} = xxx + \dot{r}_3 u_1 + r_3 z_1$$

$$= xxx + \underbrace{\dot{R}\begin{bmatrix}0\\0\\1\end{bmatrix}} u_1 + R\begin{bmatrix}0\\0\\1\end{bmatrix} z_1$$

$$= xx + R\hat{\omega}\begin{bmatrix}0\\0\\1\end{bmatrix} u_1 + R\begin{bmatrix}0\\0\\1\end{bmatrix} z_1$$

$$= xx - \underbrace{R u_1 \widehat{\begin{bmatrix}0\\0\\1\end{bmatrix}}} \omega + R\begin{bmatrix}0\\0\\1\end{bmatrix} z_1$$

$$\begin{bmatrix} y_1^{(3)} \\ y_2^{(3)} \\ y_3^{(3)} \\ y_4^{(2)} \end{bmatrix} = xx \underbrace{- R u_1 \widehat{\begin{bmatrix}0\\0\\1\end{bmatrix}} \omega} + \underbrace{\begin{bmatrix} r_3 & 0 \\ 0 & a_{42}\; a_{43}\; a_{44} \end{bmatrix}}_{\text{Still Rank 2}} \begin{array}{l} z_1 \\ u_2 \\ u_3 \\ u_4 \end{array}$$

Now differentiate again $\underbrace{\dot{z}_1 = v_1}$

$$\begin{bmatrix} y_1^{(4)} \\ y_2^{(4)} \\ y_3^{(4)} \end{bmatrix} = xxx - u_1 R \widehat{\begin{bmatrix}0\\0\\1\end{bmatrix}} \dot{\omega} + R\begin{bmatrix}0\\0\\1\end{bmatrix} v_1$$

$$= xxx - u_1 R \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} I^{-1}\left(-\omega \times I\omega + \begin{bmatrix} u_2 \\ u_3 \\ u_4 \end{bmatrix}\right)$$

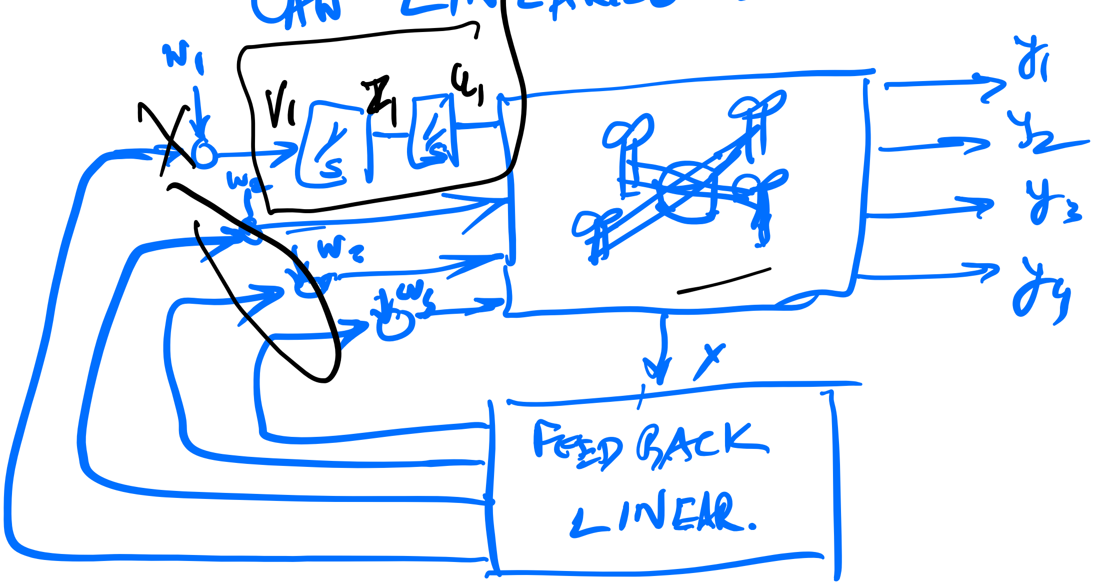
$$+ R \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} v_1$$

Now

$$\begin{bmatrix} y_1^{(4)} \\ y_2^{(4)} \\ y_3^{(4)} \\ y_4^{(2)} \end{bmatrix} = xxx + \begin{bmatrix} R\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} & -Ru_1 \begin{bmatrix} 0 & +1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ 0 & a_{42} \quad a_{43} \quad a_{44} \end{bmatrix} \begin{bmatrix} v_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}$$

If $u_1 \neq 0$ $a_{44} \neq 0$ $\Rightarrow$ Rank $= 4$

CAN LINEARIZE & DECOUPLE



RESULTS IN

$$\left.\begin{aligned} y_1^{(4)} &= w_1 \\ y_2^{(4)} &= w_2 \end{aligned}\right\} \text{POSITION}$$

$$\left.\begin{aligned} y_3^{(4)} &= w_3 \\ y_4^{(2)} &= w_4 \end{aligned}\right\} \text{YAW}$$