Today's Agenda:

1. Optimization notation
2. Model predictive control (MPC)
3. Constrained MPC

OPTIMIZATION NOTATION:

"INFORMALLY": An optimization problem seeks to find the BEST VALUE of a decision variable, x, subject to some constr!!

"Cost function" => func. of the decision variable! $f(x)$, $J(x)$

"x is good" -> $f(x)$ will be small
"x is bad" -> $f(x)$ will be large!!

Notation:

$$x^* = \underset{x \in D}{\arg\min}\ f(x)$$

"argument" ↓ (arg); min: looking for MINIMUM of $f(x)$ over all values of x in some domain D. $x \in D$

x^*: Actual optimal value!!

$$\text{s.t.}\quad g(x) \le b, \quad a(x) = c$$

} Constraints!: things that x MUST satisfy!! Constr. must be a func. of x!!

$\le$ "inequality constr"
$=$ "equality constr"

MODEL PREDICTIVE CONTROL: :)

Q: Can we formulate an opti problem to solve BOTH control AND path planning??

Setup: Suppose we have a discrete time NL sys!

$$x(k+1) = f(x(k), u(k)) \quad \text{(discretise CT as needed)}$$

$$\updownarrow$$

$$x_{k+1} = f(x_k, u_k)$$

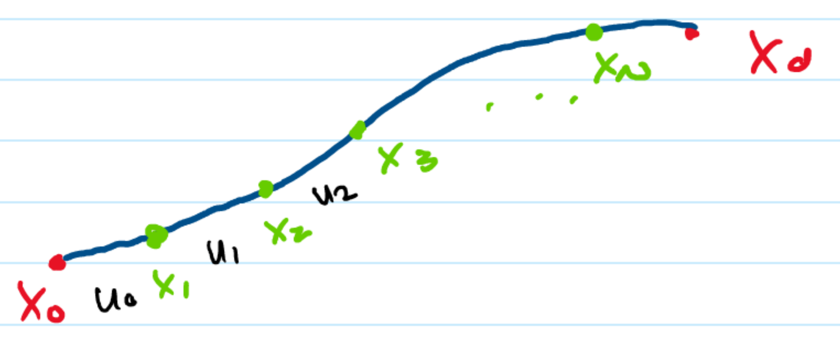
- let's start at an init cond $x(0)$, drive sys. to a DESIRED state x_d.

Q: Find an optimal path of N+1 steps:

$x_0, x_1, \ldots, x_N$ } want $x_N \sim x_d$.

& An assoc. seq. of inputs:
$u_0, u_1, \ldots, u_{N-1}$

=> that allow us to follow the path



Step 1: Decision Vars:

x: $x_0, x_1, \ldots, x_N$
u: $u_0, u_1, \ldots, u_{N-1}$ } want these!

Step 2: COST FUNCTION:

=> Energy term: $u_i^T R u_i$: $R \succeq 0$ pos. semidef.
R is symm. & has ALL eig. ≥ 0.

$u_i^T R u_i$ ~> weighted version of ☆ $\|u_i\|^2$ ☆

$$R = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \qquad u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$u^T R u = \lambda_1 u_1^2 + \lambda_2 u^2$$

=> Distance: $(x_i - x_d)^T Q (x_i - x_d)$ $Q \succeq 0$ pos. semidef.
=> Min. for $x_i = x_d$!!

=> Final distance: $(x_N - x_d)^T P (x_N - x_d)$ $P \succeq 0$
↑ final term => like to take separately from rest of seq!!

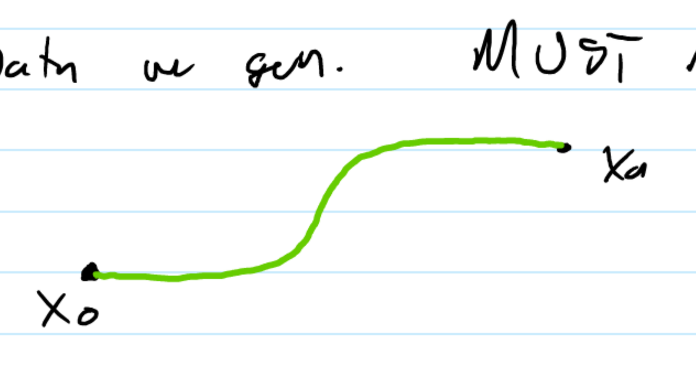
Add them up!

$$J = \underbrace{(x_N - x_d)^T P (x_N - x_d)}_{\text{"terminal cost"}} + \sum_{k=0}^{N-1} \left[(x_k - x_d)^T Q (x_k - x_d) + u_k^T R u_k \right]$$

↑ Sum over EVERY step in path!!

CONSTRAINTS:

=> Whatever path we gen. MUST respect sys. dyn!!



$x_0, x_1, \ldots, x_N \quad u_0, \ldots, u_{N-1}$

$$\text{s.t.} \quad x_{k+1} = f(x_k, u_k), \quad 0 \le k \le N-1$$

$$x_0 = x(0)$$

☆ MPC CONTROLLER: ☆

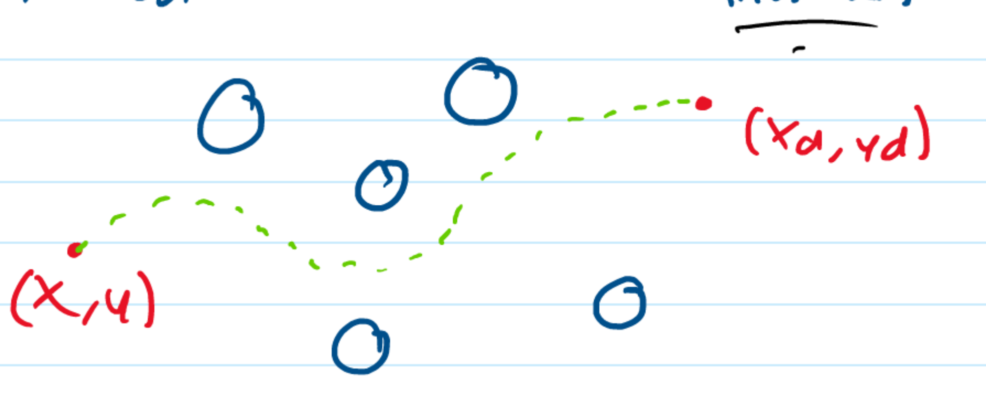
1) Feed in current sys. state as the init. cond. in opti.
2) Solve opti. problem. X, U } Computer "CASADI"
3) Send FIRST INPUT in U to the system.

Problem 1: What are (+)/(−) of high N?
Why don't we execute the whole U seq?

CONSTRAINED MPC:

- MPC decision vars: X, U
  - Directly constrain values of X & U in our opti!!

Problem 3: Suppose there are p circular obstacles btwn. our turtlebot position (x, y) & our GOAL pos, (x_d, y_d). Each obstacle has center (x_i, y_i) & radius r_i.



Find an expr. for a CONSTR. on turtlebot (x, y) s.t. the turtlebot AVOIDS all obstacles!!

Solution: $\|q_t - q_{oi}\| > r_i$
↑ obst. i.

$$(x - x_i)^2 + (y - y_i)^2 > r_i^2$$