Multiple-View Reconstruction from Scene Knowledge

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“Mathematics is the art of giving the same name to different things.”

– Henri Poincare
Symmetry is ubiquitous in man-made or natural environments
INTRODUCTION: Scene knowledge and symmetry

Parallelism (vanishing point)  Orthogonality

Congruence  Self-similarity
INTRODUCTION: Wrong assumptions

Ames room illusion  Necker’s cube illusion
INTRODUCTION: Related literature

**Mathematics:** Hilbert 18\textsuperscript{th} problem: Fedorov 1891, Hilbert 1901, Bieberbach 1910, George Polya 1924, Weyl 1952

**Cognitive Science:** Figure “goodness”: Gestalt theorists (1950s), Garner’74, Chipman’77, Marr’82, Palmer’91’99…

**Computer Vision** (isotropic & homogeneous textures): Gibson’50, Witkin’81, Garding’92’93, Malik&Rosenholtz’97, Leung&Malik’97…

**Detection & Recognition** (2D & 3D): Morola’89, Forsyth’91, Vetter’94, Mukherjee’95, Zabrodsky’95, Basri & Moses’96, Kanatani’97, Sun’97, Yang, Hong, Ma’02

**Reconstruction** (from single view): Kanade’81, Fawcett’93, Rothwell’93, Zabrodsky’95’97, van Gool et.al.’96, Carlsson’98, Svedberg and Carlsson’99, Francois and Medioni’02, Huang, Yang, Hong, Ma’02,03
Multiple-View Reconstruction from Scene Knowledge

SYMMETRY & MULTIPLE-VIEW GEOMETRY
  • Fundamental types of symmetry
  • Equivalent views
  • Symmetry based reconstruction

MULTIPLE-VIEW MULTIPLE-OBJECT ALIGNMENT
  • Scale alignment: adjacent objects in a single view
  • Scale alignment: same object in multiple views

ALGORITHMS & EXAMPLES
  • Building 3-D geometric models with symmetry
  • Symmetry extraction, detection, and matching
  • Camera calibration

SUMMARY: Problems and future work
• Why does an image of a symmetric object give away its structure?

• Why does an image of a symmetric object give away its pose?

• What else can we get from an image of a symmetric object?
Equivalent views from rotational symmetry

\[ g_0 = (R_0, T_0) \]

\[ g' = (R', T') \]
Equivalent views from reflectional symmetry

$g' = (R', T')$

$I_1 \rightarrow I'_1$
Equivalent views from translational symmetry
Definition. A set of 3–D features $S$ is called a symmetric structure if there exists a nontrivial subgroup $G$ of $E(3)$ that acts on it such that for every $g$ in $G$, the map

$$g \in G : S \rightarrow S$$

is an (isometric) automorphism of $S$. We say the structure $S$ has a group symmetry $G$.

$$X = [X, Y, Z, 1]^T \in \mathbb{R}^4, \quad x = [x, y, z]^T \in \mathbb{R}^3$$

$$g_0 = \begin{bmatrix} R_0 & T_0 \\ 0 & 1 \end{bmatrix} \in \mathbb{R}^{4\times4}, \quad \Pi_0 = [I, 0] \in \mathbb{R}^{3\times4}$$

$$x \sim \Pi_0 g_0 X \quad \rightarrow \quad g(x) \sim \Pi_0 g_0 g X$$
GEOMETRY FOR SINGLE IMAGES – Multiple “Equivalent” Views

\[ g(x) \sim \Pi_0 g_0 g X = \Pi_0 g_0 g g_0^{-1} g_0 X \]

\[ g_1(x) \sim \Pi_0 g_0 g_1 g_0^{-1} (g_0 X), \]
\[ g_2(x) \sim \Pi_0 g_0 g_2 g_0^{-1} (g_0 X), \]
\[ \vdots \]
\[ g_m(x) \sim \Pi_0 g_0 g_m g_0^{-1} (g_0 X). \]

\[ g = (R, T), \quad g' = g_0 g g_0^{-1} \]

\[ g' : \begin{cases} R' &= R_0 R R_0^T \in O(3), \\ T' &= (I - R') T_0 + R_0 T \in \mathbb{R}^3 \end{cases} \]
GEOMETRY FOR SINGLE IMAGES – Symmetric Rank Condition

\[ g_0 = (R_0, T_0) \]

\[ M = \begin{bmatrix}
    g_1(x) R'_1 x & g_1(x) T'_1 \\
    g_2(x) R'_2 x & g_2(x) T'_2 \\
    \vdots & \vdots \\
    g_m(x) R'_m x & g_m(x) T'_m 
\end{bmatrix} \]

\[ \text{rank}(M) = 1 \]

Solving \( g_0 \) from Lyapunov equations:

\[ g'_i g_0 - g_0 g_i = 0, \ i = 1, \ldots, m \]

with \( g'_i \) and \( g_i \) known.
THREE TYPES OF SYMMETRY – Reflective Symmetry

\[ \begin{cases} R' = R_0 RR^T_0 \in O(3), \\ T' = (I - R')T_0 \in \mathbb{R}^3. \end{cases} \]

\[ R'R_0 - R_0 R = 0. \]

\[ \dim(Ker(L)) = 5. \]

\[ R_0 \in Ker(L) \cap SO(3) = SO(2). \]

\[ T_0 \in (I - R')^\dagger T' + Null(I - R'). \]

\[ \dim(Null(I - R')) = 2. \]
THREE TYPES OF SYMMETRY – Rotational Symmetry

\[
\begin{align*}
R' & \doteq R_0 R R_0^T \in O(3), \\
T' & \doteq (I - R')T_0 \in \mathbb{R}^3.
\end{align*}
\]

\[R' R_0 - R_0 R = 0.\]

\[\text{dim}(\text{Ker}(L)) = 3.\]

\[R_0 \in \text{Ker}(L) \cap SO(3) = SO(2).\]

\[T_0 \in (I - R')^\dagger T' + \text{Null}(I - R').\]

\[\text{dim}(\text{Null}(I - R')) = 1.\]
THREE TYPES OF SYMMETRY – Translatory Symmetry

\[
\begin{align*}
R &= R' = I, \\
T' &= R_0 T.
\end{align*}
\]

\[R_0 T = T'.\]

\[R_0 \in SO(2), \quad T_0 \in \mathbb{R}^3.\]
<table>
<thead>
<tr>
<th>Ambiguity</th>
<th>dim(\text{Ker}(L))</th>
<th>(g_0) (general scene)</th>
<th>(g_0) (planar scene)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reflective</td>
<td>5-dimensional</td>
<td>((1+2))-parameter*</td>
<td>((0+1))-parameter</td>
</tr>
<tr>
<td>Rotational</td>
<td>3-dimensional</td>
<td>((1+1))-parameter</td>
<td>((1+0))-parameter</td>
</tr>
<tr>
<td>Translational</td>
<td>9-dimensional</td>
<td>((1+3))-parameter</td>
<td>((0+2))-parameter</td>
</tr>
</tbody>
</table>

“(a+b)–parameter” means there are an \(a\)–parameter family of ambiguity in \(R_0\) of \(g_0\) and a \(b\)–parameter family of ambiguity in \(T_0\) of \(g_0\).
Symmetry-based reconstruction (reflection)

**Reflectional symmetry**

\[ g = (R, 0) \]

\[ R = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \]

**Virtual camera–camera**

\[ g' = (R', T') \]

\[ R' = R_0 R R^T_0 \]

\[ T' = (I - R') T_0 \]
Symmetry-based reconstruction

**Epipolar constraint**

\[ g(x)^T E x = 0 \]

\[ E = \hat{T}' R' \]

**Homography**

\[ \hat{g}(x) H x = 0 \]

\[ H = R' + \frac{1}{d} T' N^T \]
Symmetry-based reconstruction (algorithm)

2 pairs of symmetric image points

Recover essential matrix or homography
\[ E = \hat{T}'R' \quad H = R' + \frac{1}{a}T'N^T \]

Decompose \( E \) or \( H \) to obtain \( \{R', T', N\} \)

Solve Lyapunov equation
\[ R'R_0 - R_0R = 0 \]

To obtain \( R_0 \) and then \( T_0 \)
Symmetry-based reconstruction (reflection)
Symmetry-based reconstruction (rotation)
Symmetry-based reconstruction (translation)
ALIGNMENT OF MULTIPLE SYMMETRIC OBJECTS

\[ g_{21} = g_2 g_1^{-1} \]

\[ d_1 = 1 \]

\[ d_2 = ? \]
Correct scales within a single image

Pick the image $x$ of a point $p$ on the intersection line

$$\lambda_1 = \frac{d_1}{N_1^T x} = \lambda_2 = \frac{d_2}{N_2^T x}$$

$$\alpha = \frac{d_2}{d_1} = \frac{N_1^T x}{N_2^T x}$$

$$g_2 \leftarrow (R_2, \alpha T_2)$$

$$g_{21} = g_2 g_1^{-1}$$
Correct scale within a single image

\[ \alpha = 0.7322 \quad \theta(N_1, N_2) = 90.36^\circ \]
Correct scales across multiple images
Correct scales across multiple images

\[
\begin{bmatrix}
    x_i - \bar{x} \\
    y_i - \bar{y}
\end{bmatrix} = \alpha \begin{bmatrix}
    u_i - \bar{u} \\
    v_i - \bar{v}
\end{bmatrix}
\]

\[i = 1, 2, 3, 4\]

\[d_2 = \alpha\]

\[g_2 \leftarrow (R_2, \alpha T_2)\]

\[g_{21} = g_2 g_1^{-1}\]
Correct scales across multiple images

\[ \alpha = 0.7433 \]
Image alignment after scales corrected
ALGORITHM: Building 3-D geometric models

1. Specify symmetric objects and correspondence
Building 3-D geometric models

2. Recover camera poses and scene structure
Building 3-D geometric models

3. Obtain 3-D model and rendering with images
APPLICATIONS – Photo Editing (Beckman Institute, UIUC)
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APPLICATIONS – Human Face Features from a Single View

INS canonical views (3/4 view)
APPLICATIONS – Symmetric Curves and Surfaces
APPLICATIONS – Unmanned Aerial Vehicles (UAVs)

Berkeley Aerial Robot (BEAR) Project

Rate: 10Hz
Accuracy: 5cm, 4°
ALGORITHM: Symmetry detection and matching

Extract, detect, match symmetric objects across images, and recover the camera poses.
Segmentation & polygon fitting

1. Color-based segmentation (mean shift)

2. Polygon fitting
Symmetry verification & recovery

3. Symmetry verification (rectangles, ...)

4. Single-view recovery
Symmetry-based matching

5. Find the only one set of camera poses that are consistent with all symmetry objects
The problem of finding the largest set of matching cells is equivalent to the problem of finding the **maximal complete subgraphs (cliques)** in the matching graph.
Camera poses and 3-D recovery

Length ratio | Reconstruction | Ground truth
---|---|---
Whiteboard | 1.506 | 1.51
Table | 1.003 | 1.00
Multiple-view matching and recovery (Ambiguities)
Multiple-view matching and recovery (Ambiguities)
ALGORITHM: Calibration from symmetry

Calibrated homography  \( H = R' + \frac{1}{d} T' N^T \)

Uncalibrated homography

\[
\tilde{H} = K (R' + \frac{1}{d} T' N^T) K^{-1}
\]

\[
\tilde{H}(KT') = KT'
\]

(vanishing point)

\[
S = K^{-1} K^{-T}
\]

\[
(KT'_1 x)^T S(KT'_1 y) = 0,
\]

\[
(KT'_2 x)^T S(KT'_2 y) = 0,
\]

\[
(KT'_1 y)^T S(KT'_2 y) = 0.
\]
Calibration with a rig is also simplified: we only need to know that there are sufficient symmetries, not necessarily the 3-D coordinates of points on the rig.
SUMMARY: Multiple-View Geometry + Symmetry

Multiple (perspective) images = multiple-view rank condition

Single image + symmetry = “multiple-view” rank condition

Multiple images + symmetry = rank condition + scale correction

Matching + symmetry = rank condition + scale correction + clique identification
SUMMARY

Multiple-view 3-D reconstruction in presence of symmetry

• Symmetry based algorithms are accurate, robust, and simple.

• Methods are baseline independent and object centered.

• Alignment and matching can and should take place in 3-D space.

• Camera self-calibration and calibration are simplified and linear.

Related applications

• Using symmetry to overcome occlusion.

• Reconstruction and rendering with non-symmetric area.

• Large scale 3-D map building of man-made environments.